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FLYING QUALITIES RESEARCH FOR HIGHLY AUGMENTED AIRCRAFT

August 1979

SEARCH DEPARTMENT

UMMAN AEROSPACE CORPORATION
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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER: RE-582
2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): FLYING QUALITIES RESEARCH FOR HIGHLY AUGMENTED AIRCRAFT
5. TYPE OF REPORT & PERIOD COVERED
6. PERFORMING ORG. REPORT NUMBER: RE-582
7. AUTHOR(s): H. T. Breul and R. C. Weston
8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Grumman Aerospace Corporation
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS
12. REPORT DATE: August 1979
13. NUMBER OF PAGES: 54
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
15. SECURITY CLASS. (of this report)

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)
18. SUPPLEMENTARY NOTES
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Aircraft Control

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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Grumman Research Department Report RE-582

FLYING QUALITIES RESEARCH

FOR

HIGHLY AUGMENTED AIRCRAFT

by

H. T. Breul

and

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System Sciences

August 1979

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ACKNOWLEDGMENT

The authors wish to thank Mr. Thomas Keller, Senior Staff Scientist, Grumman Research Department, whose valuable counsel was sought and freely given at virtually every stage of the work.

ABSTRACT

An introductory, single-degree-of-freedom experimental study of aircraft roll control was performed to demonstrate the feasibility of doing flying qualities research via direct manipulation of an aircraft's step response on a flight simulator. This is a very attractive approach for studying the flying qualities requirements of modern aircraft that will incorporate new and sophisticated digital fly-by-wire control schemes for coordinating all control forces to produce nonclassical responses to pilot commands. Central to the work was the development of a digital simulation technique, based on the mathematical concept of convolution, that provides the means for easy manipulation of the simulated vehicle's step-response characteristics. Two experiments were performed that examined several basic characteristics of an aircraft's roll-rate step-response in a simulated tracking task.

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LIST OF SYMBOLS

C	Number of configuration in a pair comparison experiment
C_j	Configuration of a pair of configurations grouped for pair comparison testing that is evaluated first
C_k	Configuration of a pair of configurations grouped for pair comparison testing that is evaluated last (second)
$E_1, E_2,$ E_3, E_4	Elements of the idealized roll rate step response, sec
E_1+E_2	Total effective time delay, sec
F	Pilot command: control stick deflection
G	System response
G_{ss}	Steady state roll rate for one inch of control stick deflection, deg/sec/in.
H	System step response
L_F	Roll control acceleration for one inch of control stick deflection
N	Number of times each pair of configurations is evaluated in a pair comparison experiment
P	Number of pairs in a pair comparison experiment
$P_{j>k}$	Proportion of pair comparison trials in which C_j was judged better than C_k
R_j, R_k	Mean pilot impression of the flying qualities of configurations C_j and C_k
ΔR_{jk}	Separation between the mean values for C_j and C_k on a scale of pilot preference
r	Coefficient of correlation

r_1, r_2	Coefficients of correlation between the pilot impressions of flying qualities of any two configurations in either Grand Experiment 1 or Grand Experiment 2
T	Total number of trials in a pair comparison experiment
t	Time
z	Standard-measure distance or deviate from the mean of a unit normal distribution
Φ	Aircraft roll angle
μsec	10^{-6} sec
σ	Standard deviation
σ_1, σ_2	Standard deviations of the distribution of pilot impressions of flying qualities for configurations in Either Grand Experiment 1 or Grand Experiment 2
τ	Time constant for linear first order roll dynamics
$\Delta\tau$	Sample interval
$(\hat{\ })$	Sample data quantity
$(\dot{\ })$	Derivative with respect to time

1. INTRODUCTION

Flight control systems of modern, high-performance military aircraft are on the brink of marked change brought about by the development of the onboard computer into a powerful, highly reliable, lightweight, and relatively inexpensive tool. With it as a primary system, dramatic new control concepts are realizable. Control configured vehicle designs can reduce aircraft weight and drag by "trading" fixed aerodynamic surfaces for artificial stabilization. Aircraft mechanical complexity can be greatly reduced by the use of wholly electronic fly-by-wire control systems. And, of particular interest here, it becomes possible to use model reference algorithms or other advanced, real time computational schemes to produce fly-by-wire commands. In this way all controls can be manipulated in whatever ways — no matter how complicated — that might be necessary to produce "ideal" aircraft responses at every flight condition.

To exploit the new capabilities fully, the control system designer needs better knowledge of what the pilot's preferences really are. This is because current flying qualities specifications and most flying qualities research start with the premise that aircraft behave in stereotyped ways. Implementation of the new control concepts can, however, change this markedly and make it possible for aircraft to respond to pilot commands in almost any way desired. Thus the old rules no longer apply, and the designer needs new guidelines for using the unprecedented freedom he will have in choosing how a vehicle responds to control inputs.

Current flying qualities research at Grumman is using some novel experimental simulation techniques in an attempt to uncover

basic pilot preferences and to provide the flying qualities specifications for the next generation of aircraft control systems. The work starts with the belief that more meaningful flying qualities experimentation is possible when the experimenter has direct control over those elements of vehicle motion that the pilot senses. Conventional system describers like pole and zero locations (though convenient to the dynamicist) are not what a pilot feels, and therefore probably are not the elements of motion that shape his basic preferences. Thus, in the present work, conventional analytic describers are abandoned, and vehicle dynamics are described by the shape of the time response to a unit step input of pilot control. For linear systems, the step response is a unique description of vehicle dynamics, and its shape graphically portrays the elements of motion that a pilot feels in direct response to his control actions. Thus, by manipulating the shape of the step response the experimenter can alter vehicle dynamics by changing those elements of vehicle motion that the pilot senses directly. In addition, the step response is a complete description of vehicle dynamics. It could represent the only item that would have to be checked for compliance to a specification, as it includes the effects of linkages, actuator performance, computational algorithms, and all other aspects of the control system between the pilot's command and the vehicle's response. Finally, as a time domain quantity, it is a better descriptor for the modern computational schemes used to produce "model reference" or "ideal" control responses.

The work described here respresents a first tentative step in a long-range program of experimental research designed to determine, from the pilot's perceptual viewpoint, what the basic

elements of a vehicle's step response are and how various combinations of them affect flyability. It is concerned primarily with the development of a technique for the ready manipulation of the step response of simulated vehicles and the demonstration of this technique as a tool for studying aircraft flying qualities. Toward this end a series of single-degree-of-freedom roll-tracking experiments were performed in which slopes, radii, and other graphical characteristics of the roll-rate step-response were the independent variables. The final goal of the program is to produce an understanding of aircraft control responses — well beyond the restrictions of old conventional aircraft stereotypes — that could be used to develop design guides for the next generation of highly augmented craft.

In this report, the Simulation Technique section covers the development of the computational algorithm for driving a simulator in a way that allows facile manipulation of the step response of the simulated vehicle. The Experimentation section includes development of the "pair comparisons" experimental technique. This procedure was instituted to accommodate the non-pilots that were used as subjects. And, lastly, the success of the methods used are demonstrated in the Discussion of Results section where the outcome of the roll-control experiments are presented.

2. SIMULATION TECHNIQUE

The central feature of the simulator studies discussed here is the computational algorithm employed to generate the simulator drive signals on a digital computer and the reasons why it was used. Classically, the dynamic behavior of a system is characterized by a differential equation and analyzed in the frequency domain via the Laplace transform. These concepts have been molded by generations of mathematicians, physicists, and engineers into extraordinarily powerful tools for insuring the successful operation of complex mechanisms. However, they have two major deficiencies when employed as system describers for flying-qualities research. First, when a pilot assumes control of an aircraft system, he becomes the final arbiter of its worth, and the researcher should therefore define system behavior in terms of elements that the pilot can relate to directly. The parameters of a differential equation or transfer function, for instance, bear no direct, simple relationship to the sensations of the operator, and it becomes difficult to establish correlation between pilot opinion or performance and the value of a parameter. A vehicle pilot does not directly sense and appreciate things like frequency, damping ratio, pole/zero location, or a stability derivative, but rather things like force on his body and cockpit motion, varying in response to his own commands. Thus, an experiment to define and quantify the dynamic elements recognized by the pilot ought to be done in the pilot's sensory domain: cockpit kinematics.

The other major deficiency of conventional analytic techniques, and the one of most concern here, is that they can become extremely complex when used to describe modern, highly augmented

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aircraft systems. To make a general study of the flying qualities of such systems requires the consideration of a very large number of parameters which, as cited above, bear no simple relationship to the sensations of the pilot. Even more discouraging is the fact that conventional analytic descriptors are very cumbersome when it comes to varying some of the simpler, more basic elements of cockpit kinematics. To make an independent variation of one characteristic of cockpit motion frequently requires the complicated adjustment of all the parameters used to describe the high-order augmented system. What is needed is a way to characterize a system by its kinematics alone by a "motion signature" that defines it exclusively and directly according to the way it moves in response to a command.

Convolution is an old mathematical process adaptable to this problem. If a system-variable's response to a particular kind of input — a step — is known, the convolution algorithm describes how that variable responds to any input. In other words, the behavior of the variable is completely defined by its step-response. The step-response becomes the desired "motion-signature" of the system, and no other descriptors are needed. Thus for flying qualities research the shape of the step-response becomes the independent variable. What we must do is mechanize the convolution integral for "real-time" solution on a digital computer in a way that allows easy, direct manipulation of the step response. Convolution is of course a linear, time-invariant process that must start with no initial conditions on the output, but this is not a limitation of any consequence in the present application. We want to study the step-response in many separate and fixed vehicle/task situations to learn what is the best for each one, and the simulation can always start from zero initial conditions. Some nonlinearities that are of interest are things like deadzone, which can be simulated by

preconditioning the command signal (stick position) and stiction or friction, which would have to be applied mechanically to the simulator's control lever.

The convolution integral can be written as

$$G(t) = F(t)\, H(o) + \int_{o}^{t} F(\tau)\, \dot{H}(t-\tau)d\tau \ldots . \qquad (1)$$

where

H	is the step response -- the "motion signature"
$\dot{H}$	is the impulse response
F	is the input or "command"
G	is the systems response to the command

The sample-data form used with uniform sampling for solution on a digital computer becomes

$$G_N = F_N H_o + \sum_{K=0}^{K=N} F_K\, \dot{H}_{N-K}\, \Delta\tau \qquad (2)$$

where $F_K = F(K\,\Delta\tau)$

$\dot{H}_K = \dot{H}(K\,\Delta\tau)$

$\Delta\tau$ = the sample interval

N = the greatest integer $\leq t/\Delta\tau$

For the applications of interest here, the step response always starts at zero; that is $H_o = 0$. Thus Eq. (2) can be simplified to

$$G_N = \sum_{K=0}^{K=N} F_K \dot{H}_{N-K}\, \Delta\tau \qquad (3)$$

This is the form used to compute system output in response to a pilot input forcing function in single-degree-of-freedom simulator tracking studies. Note that $\dot{H}$ must go to zero in a finite number of samples so that it can be stored in a finite length buffer. However, included in this class of response are all the responses of

interest here*. Although the impulse response, $\dot{H}$, is the quantity actually used to compute system output, it is still the shape of H, the roll rate step response, that is regarded as the "motion signature" and independent variable. This is somewhat arbitrary, of course. It was chosen this way, however, because intuitively it seems easier to think of the control time history as a result of the pilot making a series of incremental steps in control rather than a series of small discrete pulses (although there are some marginally stable control situations that do elicit a pulsed kind of control behavior from the pilot). This means that the independent variables in the experiment will be described as elements of the step response, while a sample-data representation of the derivative of the step response (the impulse response) will be used in the digital computer for the real-time computation of system output.

An important feature of the particular algorithm used is the speed with which the system response is generated following the sampling of a new input. The computer is triggered every $\Delta\tau$ seconds (the same $\Delta\tau$ as between the samples of $\dot{H}$). Between pulses, several things must be accomplished: the current input must be sampled; the sum of all the products of F_K and $\dot{H}_{N-K}$ must be formed and output as an updated system response, G_N; and various bookeeping tasks and buffer manipulations must be performed. Equation (4a-g) demonstrates how, despite these requirements, a very fast system response is achieved. They are expansions of Eq. (3) at successive increments in time for $o \le t \le m\Delta\tau$, where $\dot{H}$ is $n\Delta\tau$ long and $m > n$. Remember that $\dot{H}_o, \dot{H}_1, \ldots, \dot{H}_n$

---

*There are ways of using an analog computer with a digital computer to simulate infinite length impulse responses and maintain control of the shape of the response on the digital computer. However, this was not necessary in the present work.

$$G_o = F_o\dot{H}_o\Delta\tau \tag{4a}$$

$$G_1 = F_1\dot{H}_o\Delta\tau + \{F_o\dot{H}_1\}\Delta\tau \tag{4b}$$

$$G_2 = F_2\dot{H}_o\Delta\tau + \{F_1\dot{H}_1 + F_o\dot{H}_2\}\Delta\tau \tag{4c}$$

$$\vdots$$

$$G_n = F_n\dot{H}_o\Delta\tau + \{F_{n-1}\dot{H}_1 + F_{n-2}\dot{H}_2 + \ldots + F_o\dot{H}_n\}\Delta\tau \tag{4d}$$

$$G_{n+1} = F_{n+1}\dot{H}_o\Delta\tau + \{F_n\dot{H}_1 + F_{n-1}\dot{H}_2 + \ldots + F_1\dot{H}_n\}\Delta\tau \tag{4e}$$

$$G_{n+2} = F_{n+2}\dot{H}_o\Delta\tau + \{F_{n+1}\dot{H}_1 + F_n\dot{H}_2 + \ldots + F_2\dot{H}_n\}\Delta\tau \tag{4f}$$

$$\vdots$$

$$G_m = F_m\dot{H}_o\Delta\tau + \{F_{m-1}\dot{H}_1 + F_{m-2}\dot{H}_2 + \ldots + F_{m-n}\dot{H}_n\}\Delta\tau \tag{4g}$$

are stored in memory. Starting at $t = 0$, $F(t)$ is sampled to get F_o, and the single product in Eq. (4a) is formed and output as G_o. We see that F_o will be needed for products with the other elements of H_n to produce the outputs G at subsequent times. These products are formed now, stored, and F_o discarded. Next, at $t = \Delta\tau$, $F(t)$ is sampled to get F_1, and the first product in Eq. (4b) is formed, added to the second product (which had been formed at $t = o$), and output as G_1. Now F_1 also will be needed to form products with elements of H_n for future outputs. These are formed and added to the appropriate products already formed with F_o so that the quantities in brackets {}, are being created ahead of time. The process continues in this manner so that at any time $t = m\Delta\tau$, see Eq. (4g), the quantity in brackets, {}, has already been computed. When F_m is obtained by sampling $F(t)$, only one multiply and one add are needed to produce the current answer, G_m. This new result is obtained quickly and output immediately.

Then F_m is multiplied by the other $\dot{H}_n$ and these products are added to the appropriate, partially computed bracketed terms that are needed for future outputs.

The technique just described results in the overall sequence of computations shown in the time-line of Figure 1, which shows the time requirements for the major computational blocks of the program and when they occur between interrupts. The sample time, that is, the elapsed time between interrupts, must be the same as the time between the samples of $\dot{H}$ stored in memory. Thus the precision with which we wish to represent $\dot{H}$ determines $\Delta\tau$, and the completion time, T_c, is determined by the number of locations in the $\dot{H}$ buffer. For the current application we chose $\Delta\tau$ quite small (6250 μsec) because we are interested in the effects of some fairly small changes in $\dot{H}$. The $\dot{H}$ buffer has 128 locations, which allow for storage of impulse responses up to 0.8 second long, which was adequate for the present work. Manipulations of $\Delta\tau$ and the buffer length are of

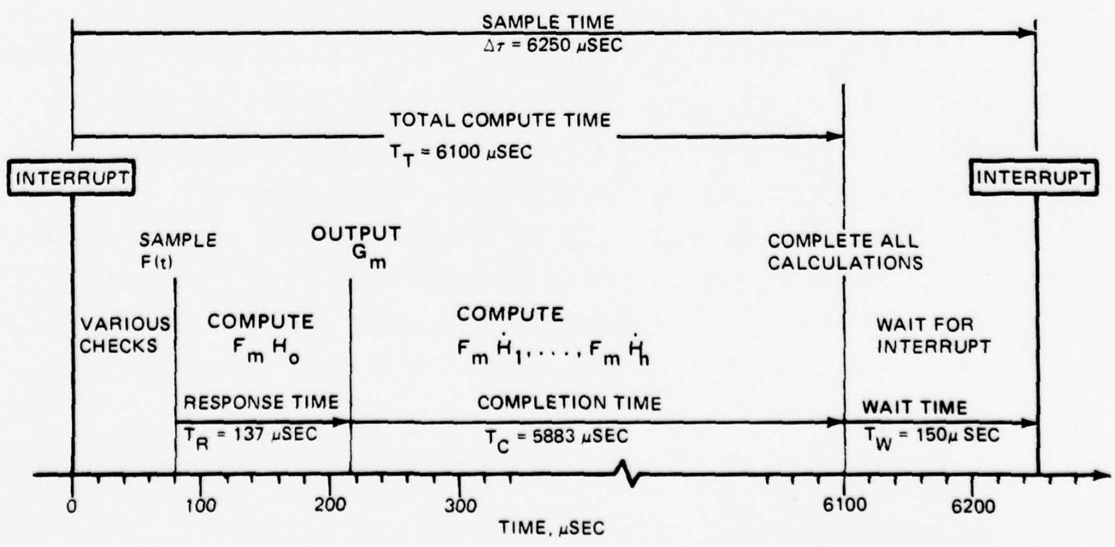

Fig. 1 Time Line Between Interrupts: Sequence of Major Events in the "Real-Time" Digital Computer Solution of the Convolution Integral

course possible to accommodate different requirements. Also, we know that the completion time, T_c, could be substantially reduced if necessary, but at the expense of using more memory. A very important feature of the present algorithm is the small and constant response time T_R (137 μsec). This is the time between sampling F(t) and producing the output G_m. If the $\dot{H}$ buffer were lengthened, forcing the completion time, T_c, to get longer, T_R would not change. (Recently, improvements have been found that will more than halve the constant T_R value.)

Impulse responses made up of straight line segments were easily "drawn" into memory using a scheme that called for typing in the coordinates of each line segment and specifying the time between samples. (Although it was not needed for the experiments discussed in this report, a more sophisticated technique is available for imputting curvilinear functions when that becomes necessary.) Once drawn, impulse responses can be transferred to magnetic tape for easy recall.

3. EXPERIMENTATION

APPROACH TO THE PROBLEM

To try out our new approach to handling qualities research employing the convolution simulation technique we chose to examine aircraft roll control in single-degree-of-freedom tracking tasks. This oft-studied, idealized control task is well suited to our needs. Aircraft roll response is typically considered to be linear and first order, and this means that there is a single dynamic parameter (and a gain) that determines both the shape of the aircraft control response and the flying qualities. It is the aircraft's roll-rate time-constant. We argue that the pilot does not sense the time constant per se and ask, "What is it about the shape of the roll response with the best time constant that the pilot finds most appealing?" Unfortunately, we did not have the resources to examine that question thoroughly in the present effort. We were, however, able to perform a few formal experiments with naive subjects in an attempt to demonstrate the validity of our approach.

For study purposes we have tentatively characterized the roll-rate step-response by four simple elements, as shown in Figure 2. The rounding described by E_4 is only one of several ways in which a step response can approach steady state. Some preliminary exploratory experiments have indicated, however, that subjects are in general more sensitive to variations in the initial part of the response described by the elements E_1,E_2, and E_3 than they are to roundings or overshoots in the final approach to a steady state value. This, and the fact of our limited resources, led us to restrict the first formal experiments to variations in E_1, E_2, E_3, and G_{ss} only, with E_4 equal to zero.

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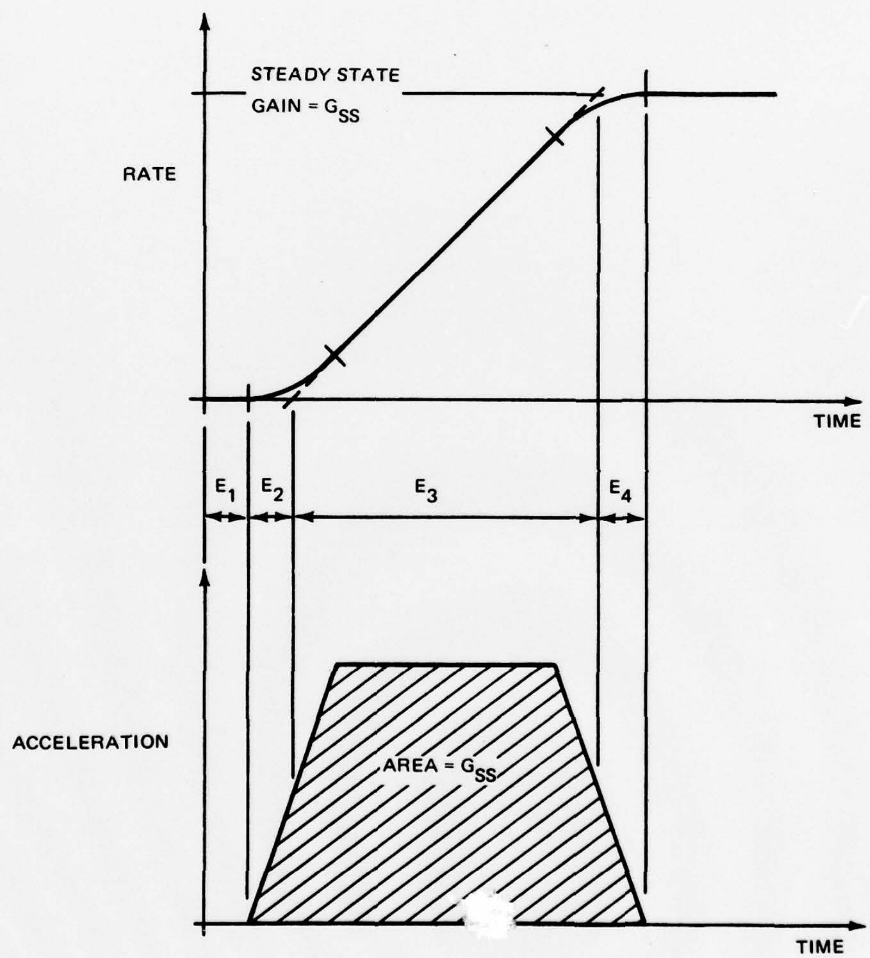


NOTE THAT WHEN $E_1 = E_2 = E_3 = E_4 = 0$, A PERFECT RATE CONTROLLER IS SIMULATED

Fig. 2 Simple, Four Element Paramaterization of Aircraft Roll Motion in Response to a Step Control Input

Two large experiments were performed. Grand Experiment 1 comprises 8 smaller experiments (numbered 1.1-1.8) in which variations in G_{ss} and E_3 are examined with $E_1 = E_2 = 0$. Grand Experiment 2 also comprises 8 smaller experiments (numbered 2.1-2.8), but with G_{ss} and E_3 equal to near-optimum values determined in Grand Experiment 1 and E_1 and E_2 varied. Both experiments were performed on a moving base simulator consisting of a chair and control stick assembly mounted on the motion platform of the Research Hover Simulator (RHS) (Fig. 3). The RHS is a six-degree-of-freedom device, but for the studies described here only the roll degree-of-freedom was active, roll motion being produced by differential driving of the two jacks supporting the motion platform on either side of the chair. Frequency response of the jacks is "flat" out to 4.5 Hz (Fig. 4). A sum of sinusoids rolling disturbance was introduced, which the subject attempted to counteract by "flying" the chair with the control stick. Figure 5 is a block diagram of this set-up. The simulator had no instruments and no special visual display. The pilot's visual scene was a view of one end of the simulation laboratory.

Stick force remained constant for both experiments. The control stick was 22 inches long and rotated in good bearings so that friction and breakout forces were not significant. Springs provided a force-feel of 0.5 pound per inch of deflection at the top of the stick.

Our choice of disturbance was somewhat arbitrary: There was available to us a digital computer subroutine that produced a pseudorandom signal as the sum of ten independent sine waves. The sinusoid frequencies were chosen to be harmonically unrelated and the amplitudes were informally adjusted to provide what was judged to be a representative disturbance in terms of pilot workload. The frequency and amplitude of the 10 sine waves are presented in Table 1.

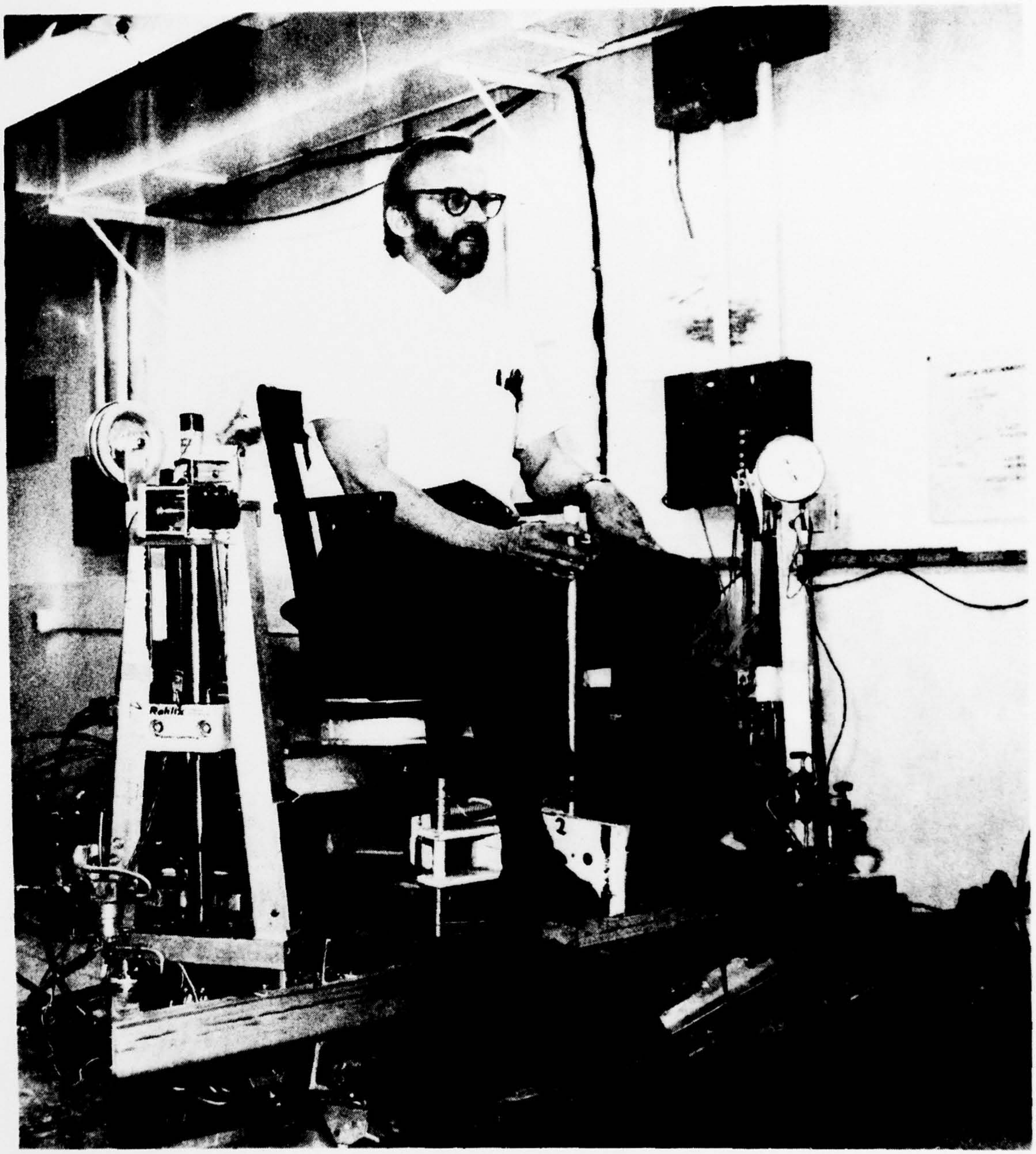


Fig. 3 Moving Base Simulator

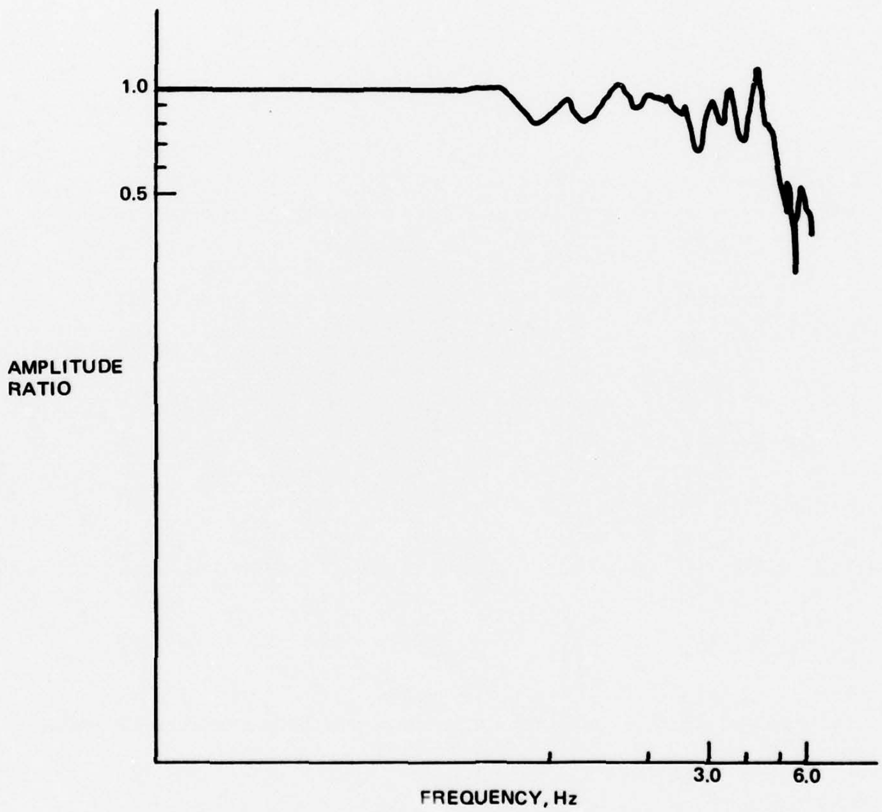


Fig. 4 Frequency Response of Moving-Base Simulator "Jacks"

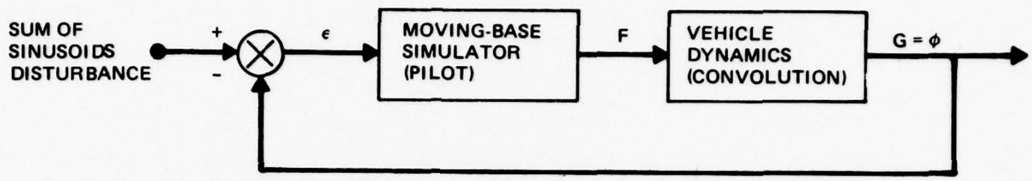


Fig. 5 Block Diagram of Moving-Base Experimental Situation

TABLE 1 SUM OF SINUSOIDS DISTURBANCE

Frequency (Hz)	Amplitude: Displacement (deg)	Amplitude: Rate (deg/sec)
0.029	3.78	0.69
0.049	↓	1.17
0.068	↓	1.61
0.107	↓	2.53
0.166	↓	3.93
0.283	↓	6.70
0.420	0.258	0.69
0.654	0.166	0.69
1.240	0.086	0.69
2.217	0.049	0.69
Std. Dev., σ	6.55	6.04

PAIR COMPARISON TESTING

To be able to perform even a small amount of meaningful experimentation within a limited budget, we adopted an approach that is unusual for modern flying qualities research. Conventional philosophy would dictate the use of a large number of subjects

who are both pilots and trained raters and who would make their evaluations using the Cooper-Harper pilot opinion scale (Ref. 1). This is admittedly desirable, but we did not have this kind of subject readily available and we could not afford to hire any. As a result the authors served as subjects, although neither is a pilot or experienced rater. This forced us to abandon conventional techniques in favor of an experimental approach more suited to the abilities of naive subjects. The most significant problem is that naive subjects cannot draw on a wealth of experience to judge the merits of a given configuration in some absolute sense. They cannot make effective use of the Cooper-Harper scale because they cannot translate their simulator experiences into real world experiences. The method adopted circumvented these difficulties. The naive subjects made only very simple judgments, but did it many many times. This method, called pair comparisons, requires only that the subject be able to judge which of a pair of configurations is the best. The method of pair comparison testing is based on Thurstone's law of comparative judgement published in 1927 (Ref. 2), and is described in the literature of experimental psychology. Guilford's text (Ref. 3) has a lucid explanation from which much of the following description is excerpted and paraphrased to fit the current application.

In the method of pair comparisons, all configurations of interest are presented to the simulator pilot in pairs, and typically all possible pairs are presented (although they need not be). In our application, the pilot "flies" each configuration through a simple tracking task on the simulator: 30 seconds with one of a pair, followed immediately by 30 seconds with the other. He then judges which of the pair he prefers -- that is, which has better flying qualities. He must pick one

even if he feels it is only a guess. Each pair is judged a large number of times, and we have as a numerical result the number and proportion of the times each configuration is preferred over every other configuration. The data are arranged in a proportion matrix such as is shown in Table 2. Each element is the proportion of the trials that the configuration at the top (identifying the column) was judged better than the configuration to the left (identifying the row). From this data we will produce for each configuration a single value on a linear scale of pilot preference.

TABLE 2 THE PROPORTION MATRIX SHOWING THE PROPORTION OF TRIALS ($P_{j>k}$) THAT EACH CONFIGURATION (C_j) IS PREFERRED OVER EVERY OTHER CONFIGURATION (C_k)

	C_a	C_b	C_c	. . .	C_j	. . .	C_n
C_a	$P_{a>a}$	$P_{b>a}$	$P_{c>a}$	. . .	$P_{j>a}$	. . .	$P_{n>a}$
C_b	$P_{a>b}$	$P_{b>b}$	$P_{c>b}$	. . .	$P_{j>b}$	. . .	$P_{n>b}$
C_c	$P_{a>c}$	$P_{b>c}$	$P_{c>c}$	. . .	$P_{j>c}$	. . .	$P_{n>c}$
.	.	.	.	. . .	.	. . .	.
C_k	$P_{a>k}$	$P_{b>k}$	$P_{c>k}$	. . .	$P_{j>k}$	. . .	$P_{n>k}$
.	.	.	.	. . .	.	. . .	.
C_n	$P_{a>n}$	$P_{b>n}$	$P_{c>n}$	. . .	$P_{j>n}$	. . .	$P_{n>n}$
$\sum$	$\sum_{k=a}^{n} P_{a>k}$	$\sum_{k=a}^{n} P_{b>k}$	$\sum_{k=a}^{n} P_{c>k}$	. . .	$\sum_{k=a}^{n} P_{j>k}$	. . .	$\sum_{k=a}^{n} P_{n>k}$

Note: The C_n's are always ordered so that the sums at the bottom increase (or decrease) monotonically from left to right.

The casual reader can skip the following details of pair comparison analysis and resume with the last paragraph on page 24.

The transformation of comparative judgement data into scale values starts with Thurstone's law of comparative judgement (Ref. 2). It is stated as follows:

$$\Delta R_{jk} = R_j - R_k = Z_{jk} \sqrt{\sigma_j^2 + \sigma_k^2 - 2r_{jk}\,\sigma_j\sigma_k} \tag{5}$$

where

R_j and R_k = mean pilot impression of the flying qualities of configurations C_j and C_k, respectively

Z_{jk} = standard-measure distance or deviate from the mean of a unit normal distribution

σ_j and σ_k = standard deviations of the distributions of R_{hj} and R_{hk}, respectively

r_{jk} = coefficient of correlation between R_{hj} and R_{hk}.

The radical term is the standard deviation of the differences $R_{hj} - R_{hk}$ and is the unit of the scale on which each separation of pilot preference $(R_j - R_k)$ is expressed. The size of each separation $R_j - R_k$ can be determined and a scale of pilot preference developed if we know the values on the right-hand side of Eq. (5). We can find Z_{jk} from the experimentally determined proportions of $P_{j>k}$, but the remaining parameters are unknown. We must, therefore, make some simplifying assumptions. We assume that the correlation coefficients are all the same and that the standard deviations are all the same. Equations (5) then becomes

$$\Delta R_{jk} = R_j - R_k = Z_{jk} \left[\sigma \sqrt{2(1-r)} \right] \tag{6}$$

The term in brackets is now a constant. It can be thought of as the unit of the scale of pilot preference and therefore set equal to unity.

Thus,

$$\Delta R_{jk} = R_j - R_k = Z_{jk} \tag{7}$$

becomes the final simplified expression.

There is of course a risk involved in making the above assumptions. The risk is minimized, however, by the fact that after we have scaled a set of configurations according to pilot preference using Eq. (7), we can check the results to see whether they are internally consistent. The test of internal consistency was developed by Mosteller (Ref. 4), and involves going backwards from the scale values calculated to produce a new proportion matrix. This would be the one expected given the scale values calculated. A chi-square test is then applied to determine the goodness of fit of the expected proportions to the original experimentally obtained proportions. If the results are internally consistent, we may say that we have found nothing to contradict the assumptions made. The test of internal consistency is elucidated in the appendix where it is applied to the data of this study.

To create a scale from the data we next use normal curve tables and Eq. (7) to transform the proportion matrix into a ΔR_{jk} matrix like the one shown in Table 3. Each ΔR_{jk} element is an estimate of the distance $R_j - R_k$, where the latter are the as yet unknown positions of configurations C_j and C_k on the scale of pilot preference. To get the best estimates of the distances between configurations we must utilize all the data. For example, ΔR_{ab} is but one estimate of the scale distance from configuration

TABLE 3 THE ΔR_{jk} MATRIX SHOWING ESTIMATES OF SEPARATIONS (ΔR_{jk}) BETWEEN EACH CONFIGURATION (C_j) AND EVERY OTHER CONFIGURATION (C_k) ON A SCALE OF PILOT PREFERENCE

	C_a	C_b	C_c	. . .	C_j	. . . C_n
C_a	ΔR_{aa}	ΔR_{ba}	ΔR_{ca}	. . .	ΔR_{ja}	. . . ΔR_{na}
C_b	ΔR_{ab}	ΔR_{bb}	ΔR_{cb}	. . .	ΔR_{jb}	. . . ΔR_{nb}
C_c	ΔR_{ac}	ΔR_{bc}	ΔR_{cc}	. . .	ΔR_{jc}	. . . ΔR_{nc}
.	.	.	.	. . .	.	. . . .
C_k	ΔR_{ak}	ΔR_{bk}	ΔR_{ck}	. . .	ΔR_{jk}	. . . ΔR_{nk}
.	.	.	.	. . .	.	. . . .
C_n	ΔR_{an}	ΔR_{bn}	ΔR_{cn}	. . .	ΔR_{jn}	. . . ΔR_{nn}
Sum of Columns	$\sum_{k=a}^{n} \Delta R_{ak}$	$\sum_{k=a}^{n} \Delta R_{bk}$	$\sum_{k=a}^{n} \Delta R_{ck}$	. . .	$\sum_{k=a}^{n} \Delta R_{jk}$	. . . $\sum_{k=a}^{n} \Delta R_{nk}$
Mean (Scale Value)	M_a	M_b	M_c	. . .	M_j	. . . M_n

C_a to configuration C_b based on a single proportion representing direct comparisons of C_a and C_b. These two configurations were also compared with all other configurations, and their relative preferences when paired with others are useful additional data. We actually have n estimates of any $(R_j - R_k)$ as shown in Eq. (8) for $(R_a - R_b)$.

$$\Delta R_{aa} - \Delta R_{ba} = R_a - R_a - R_b + R_a = (R_a - R_b) \qquad (8a)$$

$$\Delta R_{ab} - \Delta R_{bb} = R_a - R_b - R_b + R_b = (R_a - R_b) \qquad (8b)$$

$$\Delta R_{ac} - \Delta R_{bc} = R_a - R_c - R_b + R_c = (R_a - R_b) \qquad (8c)$$

$$- \quad - - \quad = - \; - - \; - - \; - - = - \; - - \qquad --$$

$$\Delta R_{an} - \Delta R_{bn} = R_a - R_n - R_b + R_n = (R_a - R_b) \qquad (8n)$$

The mean of the estimates can be obtained by averaging the column of $(R_a - R_b)$ at the far right. Note that the two columns of ΔR_{jk} at the far left of Eq (8) are identical with the first two columns of the ΔR_{jk} matrix in Table 3. Since the difference between the means is equal to the mean of the differences, we can arrive at the same end result by summing the columns first and then finding the two means. We can do this for all the columns in the ΔR_{jk} matrix, and since the differences between neighboring pairs of configurations are mean estimates of their appropriate separations, the means themselves will serve as scale values.

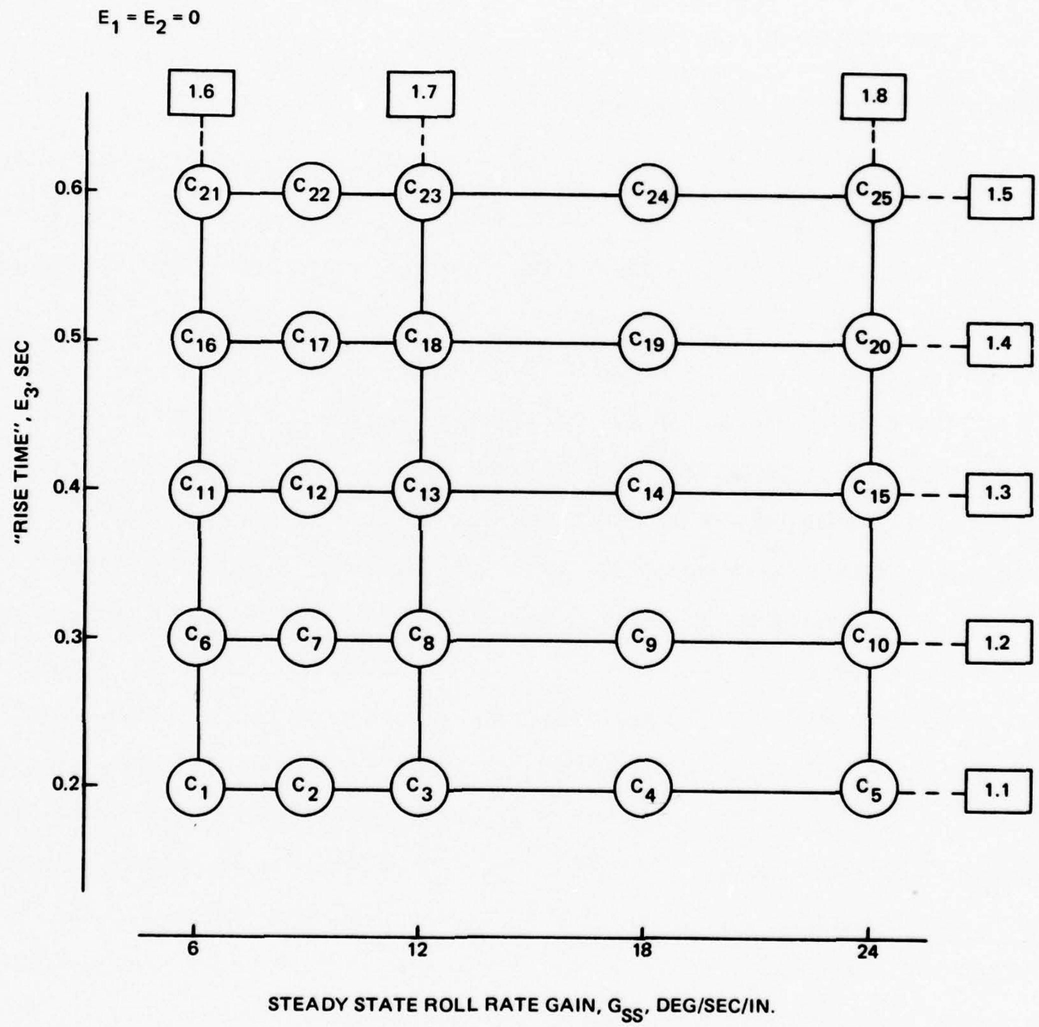
Thus we arrive at a scale of pilot preferences for the configuration compared one to another. Pair comparison testing is not without its drawbacks. The principal one arises from the fact that it provides no information about a configuration in any absolute sense, but only in terms of the differences between configurations. The scale produced is an interval or equal-unit scale where equal numerical distances represent equally distant pilot preferences, but where the zero point is arbitrary. In certain instances this could make it difficult to compare the results of different pair comparison tests. Another criticism is that repeating the same tracking task over and over to create all the elements of the proportion matrix is a boring task in

which the subjects must strive to remain alert. Despite these drawbacks, pair comparison testing proved to be satisfactory for this preliminary work, and it allowed us to make effective use of the experimenters as subjects.

EXPERIMENTAL DESIGN AND PROCEDURE

To demonstrate the utility of the convolution algorithm as a means for manipulating aircraft control responses in handling qualities research, two large experiments were performed: Grand Experiment 1 and Grand Experiment 2. Each comprised eight small pair comparison experiments (Experiments 1.1, 1.2, ..., 1.8 and Experiments 2.1, 2.2, ..., 2.8) in which a single element of aircraft roll response was varied. The experimental spaces for the two large experiments are shown in Figures 6 and 7, and within each space the various pair comparison experiments are indicated. Grand Experiment 1 was designed as a first exploratory look at some details (G_{ss} and E_3) of a response that is generally first-order in character and with no control system anomalies (such as time delay). Conversely, Grand Experiment 2 was designed as a first exploratory look at the relative effects of two kinds of control system anomalies (E_1 and E_2) when the general first-order like response is near the optimum one determined from Grand Experiment 1. Table 4 is a summary of the 50 configurations that were examined in the two grand experiments.

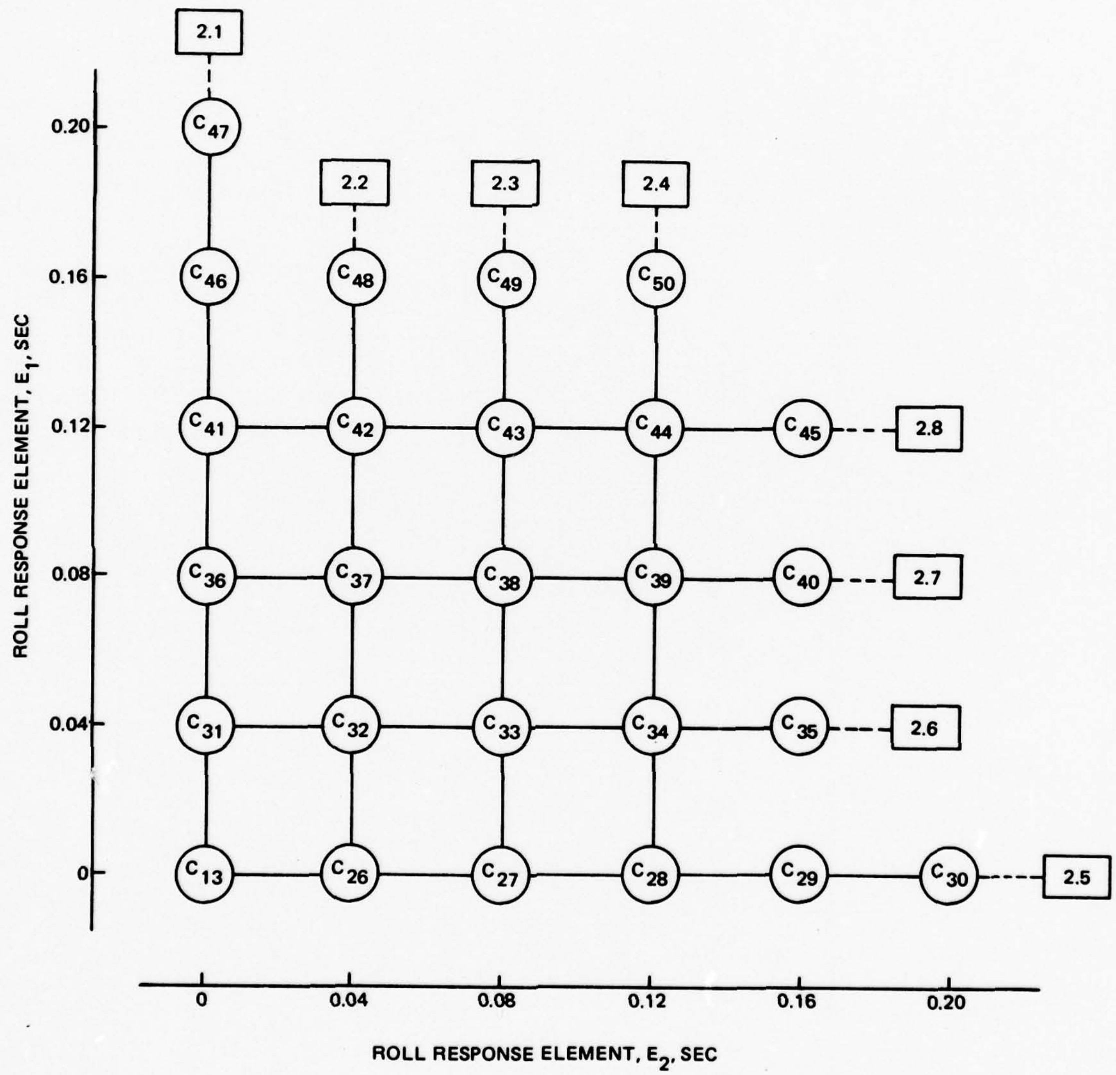
Figures 6 and 7 reveal an important strategy used to reduce the effective size of the grand experiments; that is, that we do not compare each configuration with every other configuration in the experimental space. Rather, each configuration is only compared with a small subset of configurations in either one or two very small pair comparison experiments (very small relative to the case where each configuration is compared with all others).



Note: Pair comparison experiments are indicated by lines connecting the circled configurations studied, and the experiment number is called out in a box at one end of the line. For instance, experiment 1.3 studied G_{SS} values of 6, 9, 12, 18 and 24 deg/sec/in. with E_3 = 0.4 sec. The C's in the circles are the configuration numbers as described in Table 4.

Fig. 6 Experimental Space for Grand Experiment 1

G_{SS} = 12 DEG/SEC/IN., E_3 = 0.4 SEC



Note: Pair comparison experiments are indicated by lines connecting configurations studied, and the experiment number is called out in a box at one end of the line. For instance, experiment 2.7 studied E_2 values of 0, 0.04, 0.08, 0.12 and 0.16 sec. with E_1 = 0.08 sec. The C's in the circles are the configuration numbers as described in Table 4.

Fig. 7 Experimental Space for Grand Experiment 2

TABLE 4 STEP RESPONSE CONFIGURATIONS STUDIED

Configurations	Elements				
	E_1 (sec)	E_2 (sec)	E_3 (sec)	E_4 (sec)	G_{ss} (deg/sec/in.)
C_1	0	0	0.2	0	6
C_2					9
C_3					12
C_4					18
C_5					24
C_6			0.3		6
C_7					9
C_8					12
C_9					18
C_{10}					24
C_{11}			0.4		6
C_{12}					9
C_{13}					12
C_{14}					18
C_{15}					24
C_{16}			0.5		6
C_{17}					9
C_{18}					12
C_{19}					18
C_{20}					24
C_{21}			0.6		6
C_{22}					9
C_{23}					12
C_{24}					18
C_{25}					24
C_{26}		0.04	0.4		12
C_{27}		0.08			
C_{28}		0.12			
C_{29}		0.16			
C_{30}		0.20			

TABLE 4 STEP RESPONSE CONFIGURATIONS STUDIED (CONT)

Configurations	Elements				
	E_1 (sec)	E_2 (sec)	E_3 (sec)	E_4 (sec)	G_{ss} (deg/sec/in.)
C_{31}	0.04	0	0.4	0	12
C_{32}		0.04			
C_{33}		0.08			
C_{34}		0.12			
C_{35}		0.16			
C_{36}	0.08	0			
C_{37}		0.04			
C_{38}		0.08			
C_{39}		0.12			
C_{40}		0.16			
C_{41}	0.12	0			
C_{42}		0.04			
C_{43}		0.08			
C_{44}		0.12			
C_{45}		0.16			
C_{46}	0.16	0			
C_{47}	0.20	0			
C_{48}	0.16	0.04			
C_{49}	0.16	0.08			
C_{50}	0.16	0.12			

For instance, Figure 7 shows that C_{34} is compared with C_{28}, C_{39}, C_{44} and C_{50} in Experiment 2.4 and compared with C_{31}, C_{32}, C_{33} and C_{35} in Experiment 2.6, but that it is not compared with any of the other 17 configurations in the experimental space. Obviously this provides for substantial savings in experimentation. It means, however, that we cannot directly place all the configurations in a Grand Experiment on a single scale of pilot preference. Instead, pilot preference scales are first obtained for each of the smaller experiments (1.1 through 1.8 or 2.1 through 2.8) and then these are adjusted to minimize the difference between scale values obtained for the same configuration in two different experiments (i.e., where two scales cross). The adjustments come in the form of constants that are added to the scales, and the set of eight constants needed to adjust the eight scales obtained in a grand experiment were chosen to minimize, in a least squares sense, the sum of all 15 (16 in Grand Experiment 2) differences in pilot preference that occur wherever two scales cross. This maneuver is valid for two reasons. First, in simplifying Thurstone's Law from Eq. (5) to Eq. (7), we have already made the assumption that all the scale separation elements (ΔR_{jk}) in a grand experiment have the same units, and the individual pair comparison experiments are viewed as the way to estimate the differences between selected subsets of configurations. Second, the pair comparison tests, as noted earlier, produce interval scales of pilot preference where the distance between configurations is prescribed, but where the whole scale can be arbitrarily shifted up or down by a constant amount. One thing is lost, however, and that is the ability to apply the test of internal consistency over a whole grand experiment. Only the individual one-dimensional

scales produced by the separate smaller pair comparison experiments can be checked for violations of the assumptions made in simplifying Thurstone's Law.

A significant part of our experimental design effort was spent on informal experimentation to determine the levels and ranges of variation of the independent variables. This has to be done very carefully when preparing to do pair comparison testing. Determining the proper range of variation is very important because any two configurations in an experiment must never be so "far apart" that one would always be judged better than the other. This would put ones and zeroes in the proportion matrix, and these extreme values cannot be used in the scaling process. Actually, it is common practice to consider all proportions greater than 0.977 (and less than 0.023) too extreme and omit them from the scale calculations (Ref. 3).

The number of levels of the independent variables is important because it determines the number of configurations (C) under study, and the number of pairs (P) increases quadratically with the number of configurations.

$$P = \frac{1}{2} C (C - 1) \qquad (9)$$

Consider also that an inherent part of pair comparison testing is repetitive evaluation of each pair. The total number of trials (T) then becomes:

$$T = \frac{1}{2} N C (C - 1) \qquad (10)$$

where: C is the number of configurations

N is the total number of times each pair of configurations is evaluated (independent of order of presentation)

and T is the total number of trials

In our work we had two subjects, and for most of the 16 pair comparison experiments each subject evaluated each pair 10 times so that N was 20 (for experiments 1.7, 2.3 and 2.7 the numbers were doubled, so that N was 40). Figure 8 is a plot of Eq. (10) for N equal to 20. It demonstrates the need to keep the number of configurations to a minimum. An increase of from 5 to 6, for instance, increases the total number of trials required by 100.

For most of the variables considered a compromise had to be reached to achieve the minimum number of configurations that could be used to cover the desired range of variation. For instance consider the levels of E_3 used in experiments 1.6, 1.7, and 1.8. Here, informal experimentation had indicated that we should examine E_3's as large as 0.6 second. To do this and keep the increment between configurations reasonably small (0.1 second) meant having as many as seven configurations if we were to examine all the smaller E_3's down to $E_3 = 0$. Despite the desire to include the very small values of E_3, economy dictated their elimination. Going from 7 to 5 configurations cut the number of trials by more than half: from 420 to 200 (see Fig. 8). We also reasoned that very small values of E_3 are hard to obtain in the real world, particularly in the highly augmented aircraft to which this work is addressed. Thus we settled on the five E_3 values used.

The selection of values for E_1 and E_2 used in Grand Experiment 2 (see Fig. 7) were also strongly affected by the results of informal experiments. They showed that the limits of controllability were being approached when the sum $E_1 + E_2$ reached 0.25 second, and that even relatively small values of either E_1 or E_2 were noticeably detrimental. The latter result led us to

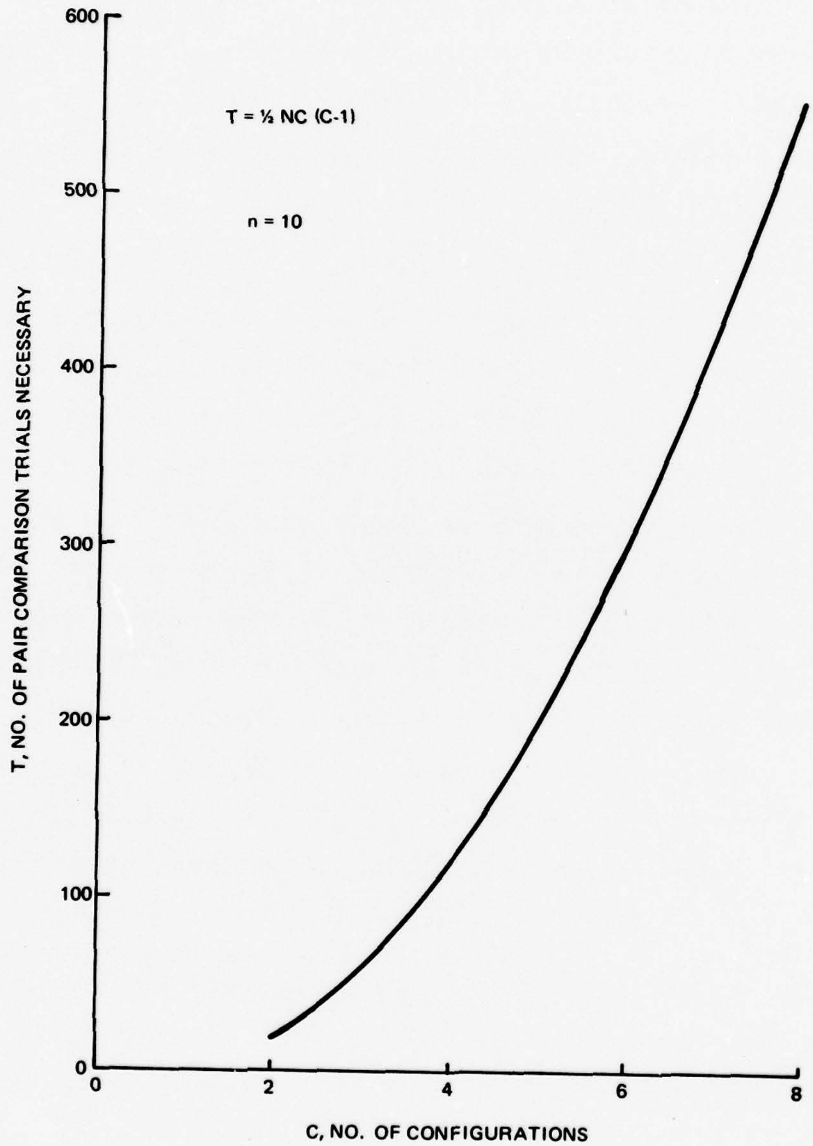


Fig. 8 Number of Trials for Pair Comparison Testing

choose 0.04 second as the increment for both E_1 and E_2. To limit the total number of trials, however, the range of variation had to be slightly compromised. E_1 and E_2 did not go above 0.16 second except in experiments 2.1 and 2.5, where values up to 0.20 second were allowed. Even so there were several configurations where $E_1 + E_2$ equaled or exceeded 0.24 second (C_{40}, C_{44}, C_{45}, C_{49} and C_{50}).

Tables 5 and 6 present the experiment matrices for Grand Experiments 1 and 2, respectively. Elements in the matrix represent pairs of configurations, and the set of pairs on one side of a diagonal is a mirror image of the set of pairs on the other side. Each pair on one side of the diagonal contains the same configurations as its mirror image on the other side, but the order of presentation is reversed. Biasing due to order of presentation was counteracted by doing the balanced design; that is, by evaluating the pairs on both sides of the diagonal an equal number of times*. We did not evaluate the pairs on the diagonal. They represent identical configurations and theoretically produce proportions of 0.5. The extent to which they do not is useful only to correct biased data from unbalanced designs.

Note that one or more pairs of configurations have been deleted from the upper right and lower left hand corners of the experimental matrices for experiments 2.1 through 2.8 (Table 6).

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*In Eq. (9), the number of pairs, P, referred to is the number of completely dissimilar pairs. That is, the number of pairs on one side of the diagonal. The total trials, T, in Eq. (10), however, include all trials independent of order of presentation.

TABLE 5 EXPERIMENT MATRICES: PAIRS OF CONFIGURATIONS* (C_j, C_k) FOR GRAND EXPERIMENT 1

C_k \ C_j	C_1	C_2	C_3	C_4	C_5
C_1	---	C_2,C_1	C_3,C_1	C_4,C_1	C_5,C_1
C_2	C_1,C_2	---	C_3,C_2	C_4,C_2	C_5,C_2
C_3	C_1,C_3	C_2,C_3	---	C_4,C_3	C_5,C_3
C_4	C_1,C_4	C_2,C_4	C_3,C_4	---	C_5,C_4
C_5	C_1,C_5	C_2,C_5	C_3,C_5	C_4,C_5	---

a) Experiment 1.1

C_k \ C_j	C_6	C_7	C_8	C_9	C_{10}
C_6	---	C_7,C_6	C_8,C_6	C_9,C_6	C_{10},C_6
C_7	C_6,C_7	---	C_8,C_7	C_9,C_7	C_{10},C_7
C_8	C_6,C_8	C_7,C_8	---	C_9,C_8	C_{10},C_8
C_9	C_6,C_9	C_7,C_9	C_8,C_9	---	C_{10},C_9
C_{10}	C_6,C_{10}	C_7,C_{10}	C_8,C_{10}	C_9,C_{10}	---

b) Experiment 1.2

C_k \ C_j	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}
C_{11}	---	C_{12},C_{11}	C_{13},C_{11}	C_{14},C_{11}	C_{15},C_{11}
C_{12}	C_{11},C_{12}	---	C_{13},C_{12}	C_{14},C_{12}	C_{15},C_{12}
C_{13}	C_{11},C_{13}	C_{12},C_{13}	---	C_{14},C_{13}	C_{15},C_{13}
C_{14}	C_{11},C_{14}	C_{12},C_{14}	C_{13},C_{14}	---	C_{15},C_{14}
C_{15}	C_{11},C_{15}	C_{12},C_{15}	C_{13},C_{15}	C_{14},C_{15}	---

c) Experiment 1.3

C_k \ C_j	C_{16}	C_{17}	C_{18}	C_{19}	C_{20}
C_{16}	---	C_{17},C_{16}	C_{18},C_{16}	C_{19},C_{16}	C_{20},C_{16}
C_{17}	C_{16},C_{17}	---	C_{18},C_{17}	C_{19},C_{17}	C_{20},C_{17}
C_{18}	C_{16},C_{18}	C_{17},C_{18}	---	C_{19},C_{18}	C_{20},C_{18}
C_{19}	C_{16},C_{19}	C_{17},C_{19}	C_{18},C_{19}	---	C_{20},C_{19}
C_{20}	C_{16},C_{20}	C_{17},C_{20}	C_{18},C_{20}	C_{19},C_{20}	---

d) Experiment 1.4

* The C_j are the configurations presented first in each pair, and the C_k are presented last. All configurations are defined in Table 4.

TABLE 5 EXPERIMENT MATRICES: PAIRS OF CONFIGURATIONS* (C_j, C_k) FOR GRAND EXPERIMENT 1 (CONT)

$C_k \backslash C_j$	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}
C_{21}	---	C_{22},C_{21}	C_{23},C_{21}	C_{24},C_{21}	C_{25},C_{21}
C_{22}	C_{21},C_{22}	---	C_{23},C_{22}	C_{24},C_{22}	C_{25},C_{22}
C_{23}	C_{21},C_{23}	C_{22},C_{23}	---	C_{24},C_{23}	C_{25},C_{23}
C_{24}	C_{21},C_{24}	C_{22},C_{24}	C_{23},C_{24}	---	C_{25},C_{24}
C_{25}	C_{21},C_{25}	C_{22},C_{25}	C_{23},C_{25}	C_{24},C_{25}	---

e) Experiment 1.5

$C_k \backslash C_j$	C_{21}	C_{16}	C_{11}	C_6	C_1
C_{21}	---	C_{16},C_{21}	C_{11},C_{21}	C_6,C_{21}	C_1,C_{21}
C_{16}	C_{21},C_{16}	---	C_{11},C_{16}	C_6,C_{16}	C_1,C_{16}
C_{11}	C_{21},C_{11}	C_{16},C_{11}	---	C_6,C_{11}	C_1,C_{11}
C_6	C_{21},C_6	C_{16},C_6	C_{11},C_6	---	C_1,C_6
C_1	C_{21},C_1	C_{16},C_1	C_{11},C_1	C_6,C_1	---

f) Experiment 1.6

$C_k \backslash C_j$	C_{23}	C_{18}	C_{13}	C_8	C_3
C_{23}	---	C_{18},C_{23}	C_{13},C_{23}	C_8,C_{23}	C_3,C_{23}
C_{18}	C_{23},C_{18}	---	C_{13},C_{18}	C_8,C_{18}	C_3,C_{18}
C_{13}	C_{23},C_{13}	C_{18},C_{13}	---	C_8,C_{13}	C_3,C_{13}
C_8	C_{23},C_8	C_{18},C_8	C_{13},C_8	---	C_3,C_8
C_3	C_{23},C_3	C_{18},C_3	C_{13},C_3	C_8,C_3	---

g) Experiment 1.7

$C_k \backslash C_j$	C_{25}	C_{20}	C_{15}	C_{10}	C_5
C_{25}	---	C_{20},C_{25}	C_{15},C_{25}	C_{10},C_{25}	C_5,C_{25}
C_{20}	C_{25},C_{20}	---	C_{15},C_{20}	C_{10},C_{20}	C_5,C_{20}
C_{15}	C_{25},C_{15}	C_{20},C_{15}	---	C_{10},C_{15}	C_5,C_{15}
C_{10}	C_{25},C_{10}	C_{20},C_{10}	C_{15},C_{10}	---	C_5,C_{10}
C_5	C_{25},C_5	C_{20},C_5	C_{15},C_5	C_{10},C_5	---

h) Experiment 1.8

*The C_j are the configurations presented first in each pair, and the C_k are presented last. All configurations are defined in Table 4.

TABLE 6 EXPERIMENT MATRICES: PAIRS OF CONFIGURATIONS* (C_j, C_k) FOR GRAND EXPERIMENT 2

C_k \ C_j	C_{47}	C_{46}	C_{41}	C_{36}	C_{31}	C_{13}
C_{47}	---	C_{46},C_{47}	C_{41},C_{47}	C_{36},C_{47}	---	---
C_{46}	C_{47},C_{46}	---	C_{41},C_{46}	C_{36},C_{46}	---	---
C_{41}	C_{47},C_{41}	C_{46},C_{41}	---	C_{36},C_{41}	C_{31},C_{41}	C_{13},C_{41}
C_{36}	C_{47},C_{36}	C_{46},C_{36}	C_{41},C_{36}	---	C_{31},C_{36}	C_{13},C_{36}
C_{31}	---	---	C_{41},C_{31}	C_{36},C_{31}	---	C_{13},C_{31}
C_{13}	---	---	C_{41},C_{13}	C_{36},C_{13}	C_{31},C_{13}	---

a) Experiment 2.1

C_k \ C_j	C_{48}	C_{42}	C_{37}	C_{32}	C_{26}
C_{48}	---	C_{42},C_{48}	C_{37},C_{48}	C_{32},C_{48}	---
C_{42}	C_{48},C_{42}	---	C_{37},C_{42}	C_{32},C_{42}	C_{26},C_{42}
C_{37}	C_{48},C_{37}	C_{42},C_{37}	---	C_{32},C_{37}	C_{26},C_{37}
C_{32}	C_{48},C_{32}	C_{42},C_{32}	C_{37},C_{32}	---	C_{26},C_{32}
C_{26}	---	C_{42},C_{26}	C_{37},C_{26}	C_{32},C_{26}	---

b) Experiment 2.2

C_k \ C_j	C_{49}	C_{43}	C_{38}	C_{33}	C_{27}
C_{49}	---	C_{43},C_{49}	C_{38},C_{49}	C_{33},C_{49}	---
C_{43}	C_{49},C_{43}	---	C_{38},C_{43}	C_{33},C_{43}	C_{27},C_{43}
C_{38}	C_{49},C_{38}	C_{43},C_{38}	---	C_{33},C_{38}	C_{27},C_{38}
C_{33}	C_{49},C_{33}	C_{43},C_{33}	C_{38},C_{33}	---	C_{27},C_{33}
C_{27}	---	C_{43},C_{27}	C_{38},C_{27}	C_{33},C_{27}	---

c) Experiment 2.3

C_k \ C_j	C_{50}	C_{44}	C_{39}	C_{34}	C_{28}
C_{50}	---	C_{44},C_{50}	C_{39},C_{50}	C_{34},C_{50}	C_{28},C_{50}
C_{44}	C_{50},C_{44}	---	C_{39},C_{44}	C_{34},C_{44}	C_{28},C_{44}
C_{39}	C_{50},C_{39}	C_{44},C_{39}	---	C_{34},C_{39}	C_{28},C_{39}
C_{34}	C_{50},C_{34}	C_{44},C_{34}	C_{39},C_{34}	---	C_{29},C_{34}
C_{28}	C_{50},C_{28}	C_{44},C_{28}	C_{39},C_{28}	C_{34},C_{28}	---

d) Experiment 2.4

* The C_j are the configurations presented first in each pair, and the C_k are presented last. All configurations are defined in Table 4.

TABLE 6 EXPERIMENT MATRICES: PAIRS OF CONFIGURATIONS* (C_j, C_k) FOR GRAND EXPERIMENT 2 (CONT)

C_k \ C_j	C_{30}	C_{29}	C_{28}	C_{27}	C_{26}	C_{13}
C_{30}	---	C_{29},C_{30}	C_{28},C_{30}	C_{27},C_{30}	---	---
C_{29}	C_{30},C_{29}	---	C_{28},C_{29}	C_{27},C_{29}	---	---
C_{28}	C_{30},C_{28}	C_{29},C_{28}	---	C_{27},C_{28}	C_{26},C_{28}	C_{13},C_{28}
C_{27}	C_{30},C_{27}	C_{29},C_{27}	C_{28},C_{27}	---	C_{26},C_{27}	C_{13},C_{27}
C_{26}	---	---	C_{28},C_{26}	C_{27},C_{26}	---	C_{13},C_{26}
C_{13}	---	---	C_{28},C_{13}	C_{27},C_{13}	C_{26},C_{13}	---

e) Experiment 2.5

C_k \ C_j	C_{35}	C_{34}	C_{33}	C_{32}	C_{31}
C_{35}	---	C_{34},C_{35}	C_{33},C_{35}	C_{32},C_{35}	---
C_{34}	C_{35},C_{34}	---	C_{33},C_{34}	C_{32},C_{34}	C_{31},C_{34}
C_{33}	C_{35},C_{33}	C_{34},C_{33}	---	C_{32},C_{33}	C_{31},C_{33}
C_{32}	C_{35},C_{32}	C_{34},C_{32}	C_{33},C_{32}	---	C_{31},C_{32}
C_{31}	---	C_{34},C_{31}	C_{33},C_{31}	C_{32},C_{31}	---

f) Experiment 2.6

C_k \ C_j	C_{40}	C_{39}	C_{38}	C_{37}	C_{36}
C_{40}	---	C_{39},C_{40}	C_{38},C_{40}	C_{37},C_{40}	---
C_{39}	C_{40},C_{39}	---	C_{38},C_{39}	C_{37},C_{39}	C_{36},C_{39}
C_{38}	C_{40},C_{38}	C_{39},C_{38}	---	C_{37},C_{38}	C_{36},C_{38}
C_{37}	C_{40},C_{37}	C_{39},C_{37}	C_{38},C_{37}	---	C_{36},C_{37}
C_{36}	---	C_{39},C_{36}	C_{38},C_{36}	C_{37},C_{36}	---

g) Experiment 2.7

C_k \ C_j	C_{45}	C_{44}	C_{43}	C_{42}	C_{41}
C_{45}	---	C_{44},C_{45}	C_{43},C_{45}	C_{42},C_{45}	---
C_{44}	C_{45},C_{44}	---	C_{43},C_{44}	C_{42},C_{44}	C_{41},C_{44}
C_{43}	C_{45},C_{43}	C_{44},C_{43}	---	C_{42},C_{43}	C_{41},C_{43}
C_{42}	C_{45},C_{42}	C_{44},C_{42}	C_{43},C_{42}	---	C_{41},C_{42}
C_{41}	---	C_{44},C_{41}	C_{43},C_{41}	C_{42},C_{41}	---

h) Experiment 2.8

* The C_j are the configurations presented first in each pair, and the C_k are presented last. All configurations are defined in Table 4.

These economies stemmed from informal experiments, which suggested that increases in E_1 or E_2 would always be detrimental. Thus we could make an educated guess about what the rank order of the configurations would be in the proportion matrices before the experiments were performed. It was assumed they would be ordered according to decreasing values of the variable (either E_1 or E_2), and that this would correspond to a monotonic increase in pilot preference. Thus it was assumed that each proportion matrix in Grand Experiment 2 would end up being ordered the same way as its respective experiment matrix. This assumption (which the experiments ultimately justified) made it possible to select the pairs of configuration that could be logically omitted from the experiments for the sake of economy. The pairs in the corners of the proportion matrices furthest from the diagonal are the ones most likely to produce proportions close to zero (or one) and are the most risky for use in producing scale values. Thus 28 pairs (15 percent of the total) were deleted from the experiment matrices for the eight experiments comprising Grand Experiment 2.

Despite this strong effort to economize, Grand Experiments 1 and 2 still remained large. A total of 3680 trials were required to complete them: 1800 for Grand Experiment 1 and 1880 for Grand Experiment 2. Even so, the experimentation went fairly rapidly. The 3680 trials were completed in about 30 days of running, over a period of several months. Because some of our laboratory facilities were shared with other programs, we would typically run for periods of 3 to 5 days. In this work, one subject served as the "pilot" while the other acted as the "experimenter," swapping roles after every 10 trials. Each kept the records of the other so that neither could gain knowledge

of his own performance prior to the completion of an experiment. The experimenter working at the console, would call up from computer memory a pair of configurations. (The computer used was an Ambilog 200 Stored Program Signal Processor made by Adage, Inc., Boston, Mass.) He would start the problem, and the pilot would begin tracking with the first configuration. After 30 seconds the disturbance would stop, and a light would start flashing at the pilot's station, signalling the end of tracking with the first configuration. The flashing continued for 4 seconds, after which tracking with the second configuration began. At this time the light stopped flashing but remained lit to remind the pilot that he was tracking with the second configuration of a pair. After another 30 seconds the disturbance stopped again and the light went off, signalling the end of the trial. The pilot indicated which configuration he preferred by pushing either of two buttons on a control box mounted on his seat. His response was displayed on a cathode ray tube at the computer console and was recorded by the experimenter who then started the cycle again by loading the next, randomly selected pair of configurations. In general, all the pairs for all the comparison experiments composing each grand experiment were put together into two large random sequences of trials. Excluded from these two lists were the pairs for experiments 1.7, 2.3, and 2.7. They were run as three separate experiments before the large groups of trials as a final check on the levels and ranges of variation of the independent variables. They were also used to verify that an N of 20 was adequate.

4. DISCUSSION OF RESULTS

The details of the data analysis for the experiments described in the previous section are reported in the appendix. Briefly, for each grand experiment a cubic polynomial in the two experimental variables was fit in a least squares sense to the pilot preference data, and the graphs presented here are mostly planar sections of these cubic surfaces. Although we discuss the impact of these results on understanding the pilot's flying qualities preferences, we stress the larger purpose of the study; to assess the feasibility of using real time convolution on a digital computer to simulate system dynamics for studying the flying qualities of highly augmented craft.

The results of Grand Experiment 1 are summarized in Figures 9 and 10. Figure 9 contains plots of pilot preference versus steady state gain, G_{ss}, at E_3 values of 0.2, 0.3, 0.4, 0.5 and 0.6 second that were determined from experiments 1.1, 1.2, 1.3, 1.4 and 1.5, respectively. The pilot preference scale on the ordinate ranges from the "worst" or least preferred configuration to the "best" or most preferred configuration of Grand Experiment 1. Values in between are regarded as being some percentage of the best; that is, a configuration with a pilot preference rating of 75 percent is said to be regarded by the pilots as being 75 percent as good as the best configuration examined. Figure 9 shows a well defined optimum steady state gain, G_{ss}, that only varies between the values of 12 and 15 deg/sec/in. over the whole range of E_3 values (0.2 sec $\lesssim E_3 \lesssim$ 0.6 sec). This small range for the optimum G_{ss} strongly suggests that it may indeed be some sort of a fundamental parameter as far as the pilot is concerned.

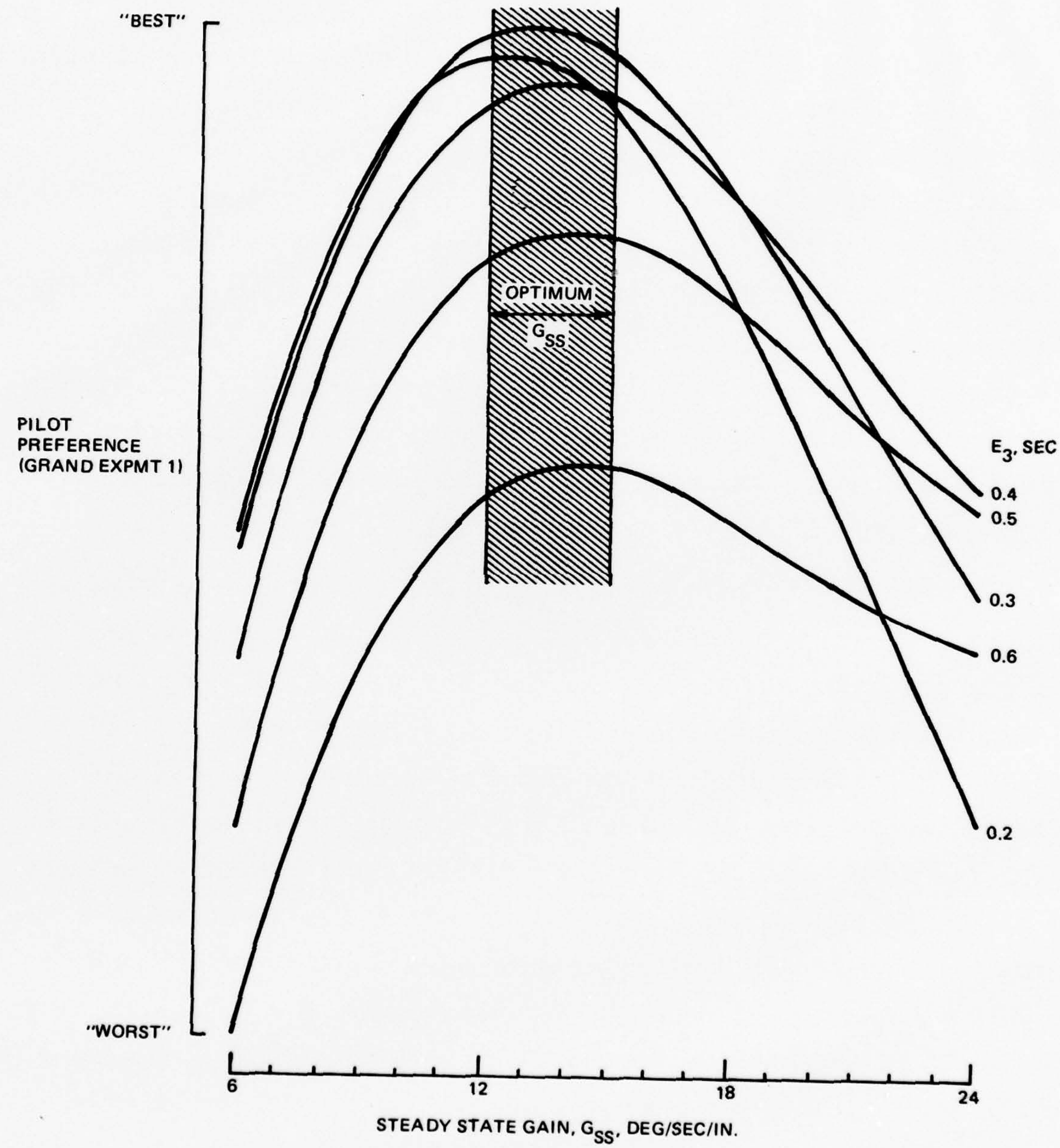


Fig. 9 Summary of Results of Grand Experiment 1: Pilot Preference versus Steady State Gain, G_{SS}

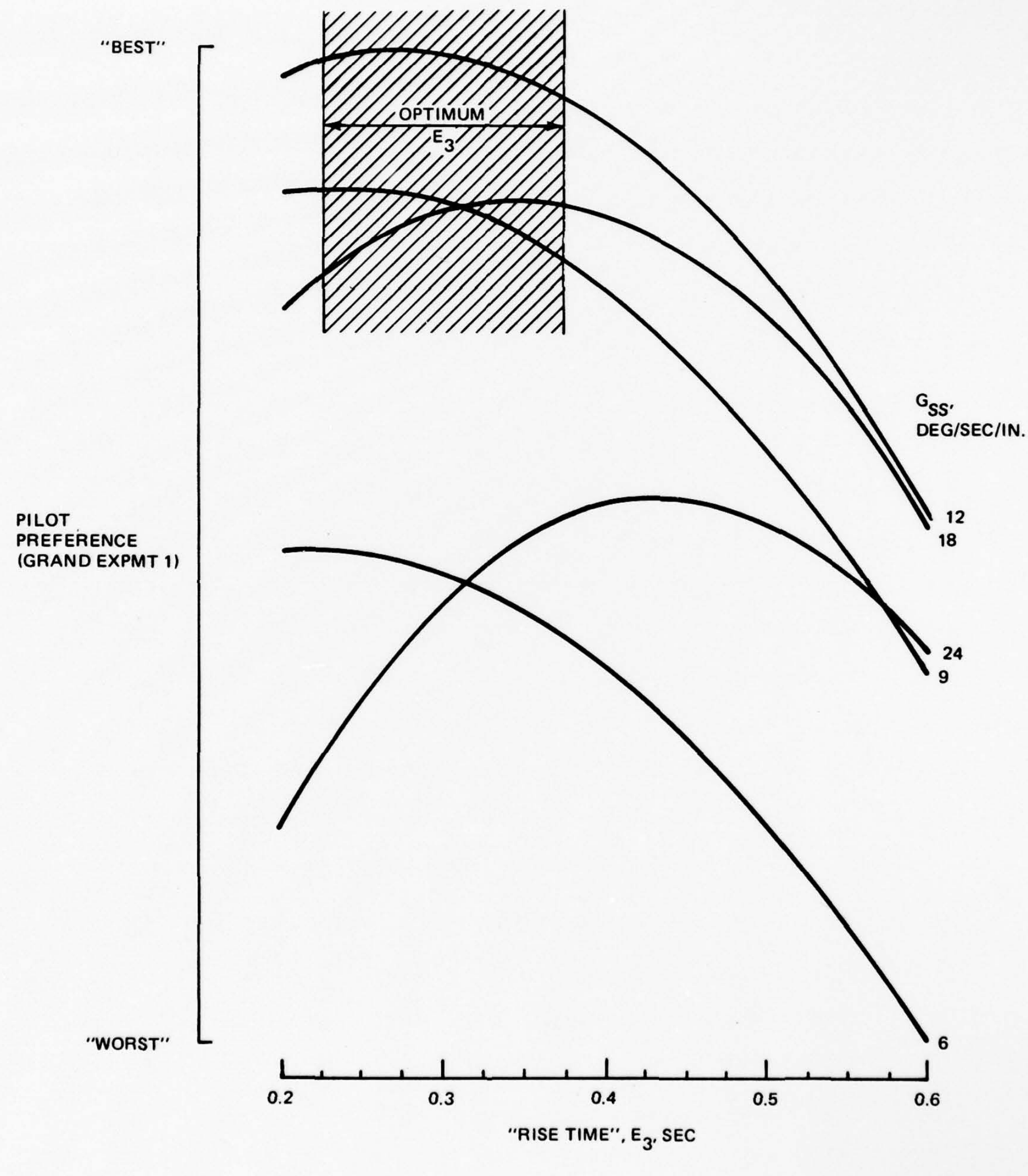


Fig. 10 Summary of Results of Grand Experiment 1: Pilot Preference versus "Rise Time", E_3

Figure 10 shows that there is also a well defined maximum "rise time," E_3, and that the fastest response (i.e., when $E_3 = 0$) is never the best. This is important because although modern control system technology might contrive a way to provide very fast responses it would no doubt come at the expense of added mechanical complexity or exorbitant control power requirements, or both. The two curves for G_{ss} equal to 9 and 18 deg/sec/in. cover the range of optimum G_{ss} values (12 to 15 deg/sec/in.) determined from Figure 9, and from them we see that the optimum E_3 value must fall within 0.225 to 0.375 second.

Although the optimum G_{ss} remains fairly constant with increasing E_3 (see Fig. 9), we see that at the highest values of E_3 (0.5 and 0.6 second) the pilot preference versus G_{ss} curves are becoming noticeably flatter. If this were the precursor to the optimum G_{ss} value increasing much more rapidly with further increases in E_3, it might indicate that there is a minimum initial acceleration (G_{ss}/E_3) wanted by the pilot independent of G_{ss} and E_3 alone. This conjecture is belied, however, by Figure 11 where we have plotted pilot preference versus G_{ss}/E_3. The minimum acceptable "acceleration" for a given pilot preference is clearly not unique.

Another interesting way to look at the results of Figures 9 and 10 is to convert them into the bounded regions of Figure 12. These delineate where the "knee" of the idealized roll-rate step-response curve must fall if the response is to achieve the specified level of pilot preference (e.g., the 90 percent region contains the "knee" of all responses that are at least 90 percent as good as the "best" response). It should be noted that the experimenter felt that only the very best of the configurations were actually good. Thus we might

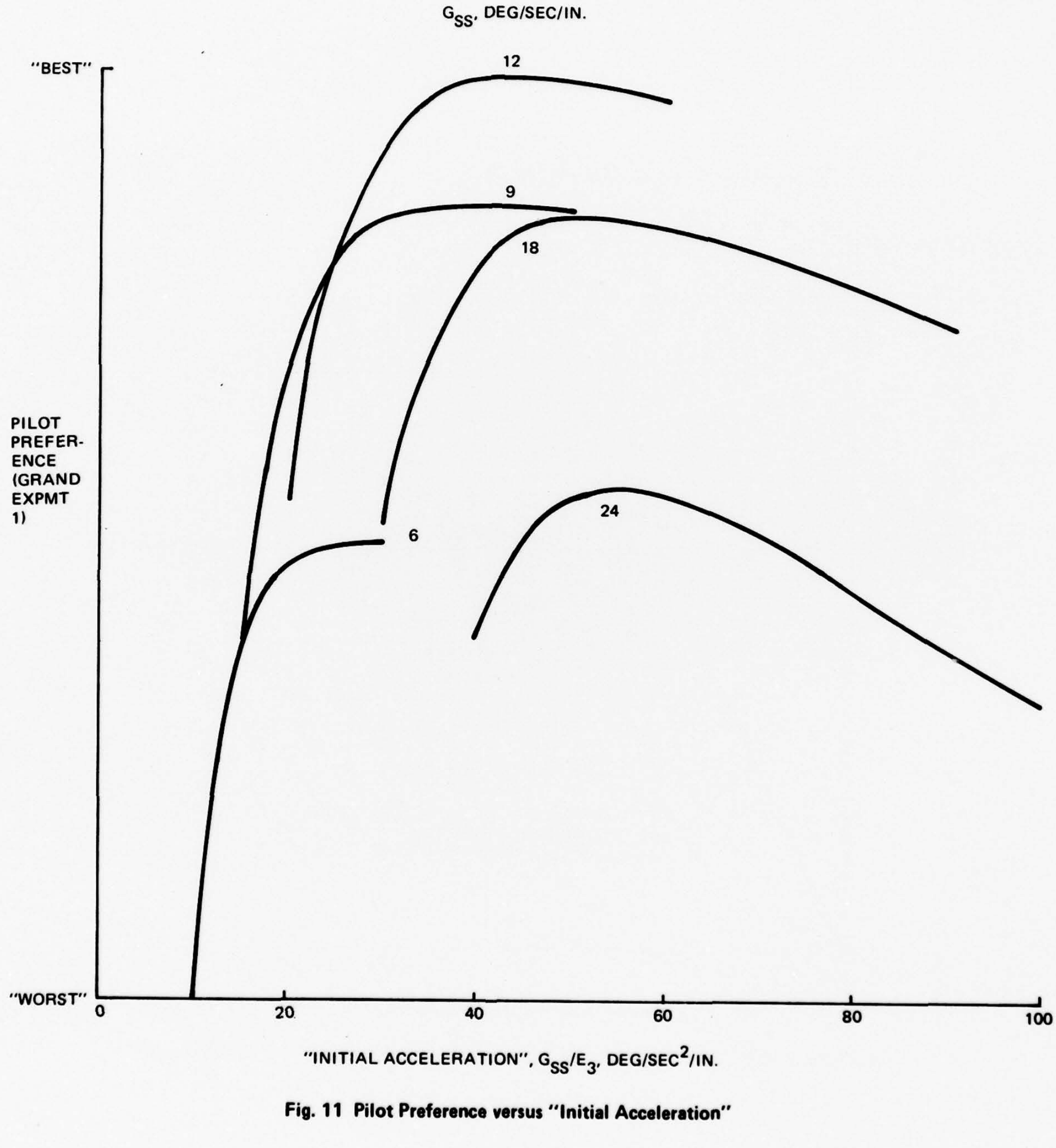


Fig. 11 Pilot Preference versus "Initial Acceleration"

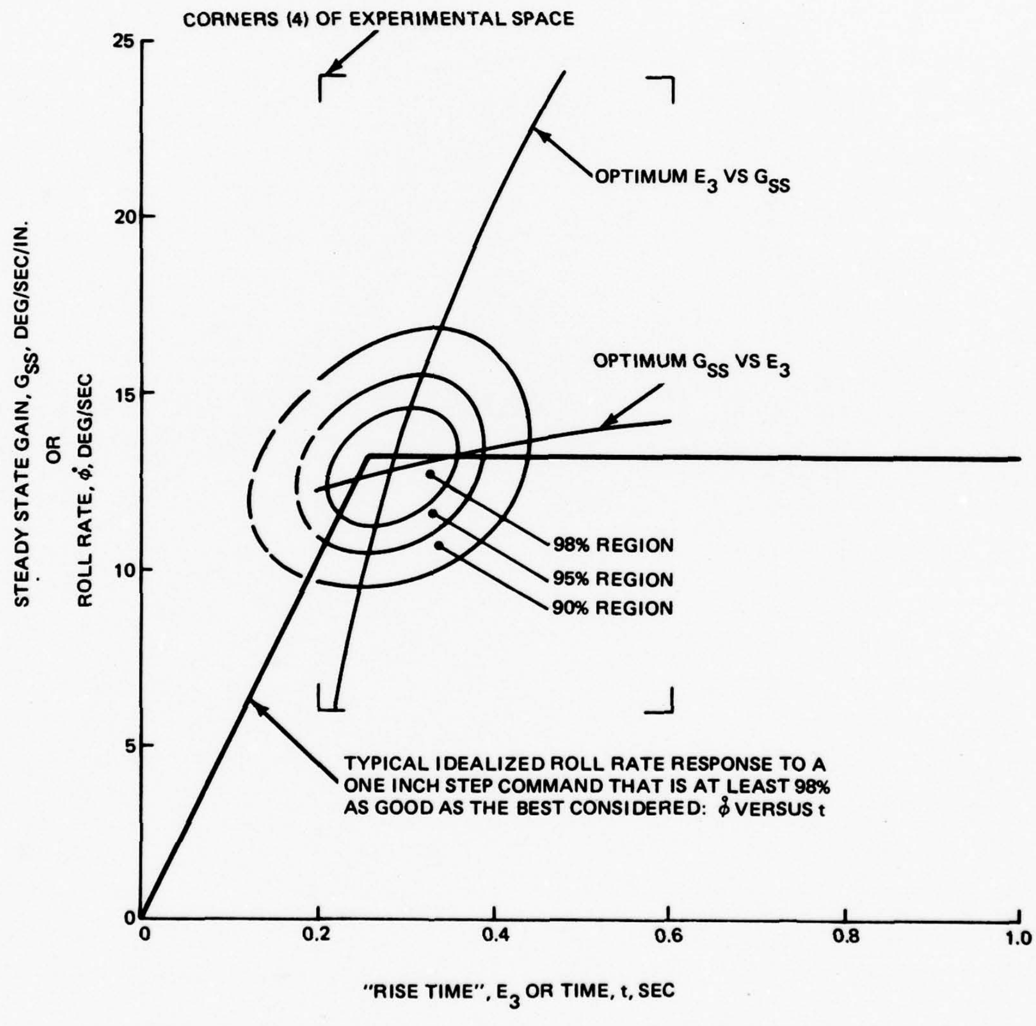


Fig. 12 Most Preferred Regions for the "Knee" of the Idealized Roll Rate Step Response

conclude that the good configurations are found only in the 95 or 98 percent regions. This presentation is really the most graphic, and it shows that for the idealized control systems being considered the best configurations (i.e., those most preferred by the pilots) have step-responses that lie in a small well defined region of the step-response plane. Note that in this figure E_3 equals t (for $0 \lesssim t \lesssim E_3$), and G_{ss} equals $\dot{\phi}$ (for $E_3 \lesssim t \lesssim \infty$). For the 95 percent region E_3 varies at most between 0.17 and 0.37 second and G_{ss} varies at most between about 10 and 15 deg/sec/in. This is of course consistent with the narrow range of optimum values for both G_{ss} and E_3 displayed by Figures 9 and 10. Also shown are a line of optimum E_3 as a function of G_{ss} and a line of optimum G_{ss} as a function of E_3. The optimum G_{ss} line is very flat and demonstrates quite clearly that the best value for G_{ss} is virtually independent of E_3; once again implying that G_{ss} is in fact a fundamental parameter that the pilot is sensitive to in his evaluation of flying qualities.

Next we would like to validate our data by comparing it with some flight test experiments performed by Princeton University (Ref. 5). For purpose of comparison we define in Figure 13 how the idealized roll rate response used here is approximated by the linear first order response used in Ref. 5 (or vice versa). The Princeton work cited here examined roll control sensitivity and rolling time constant in a variable-stability airplane that was flown through a landing approach. Flights were made in calm air with a "moderate" level of turbulence, "such as might be present near a squall line," imposed on the vehicle by manipulation of control surfaces through the autopilot. Thus the flight task was quite similar to the one used in our simulator

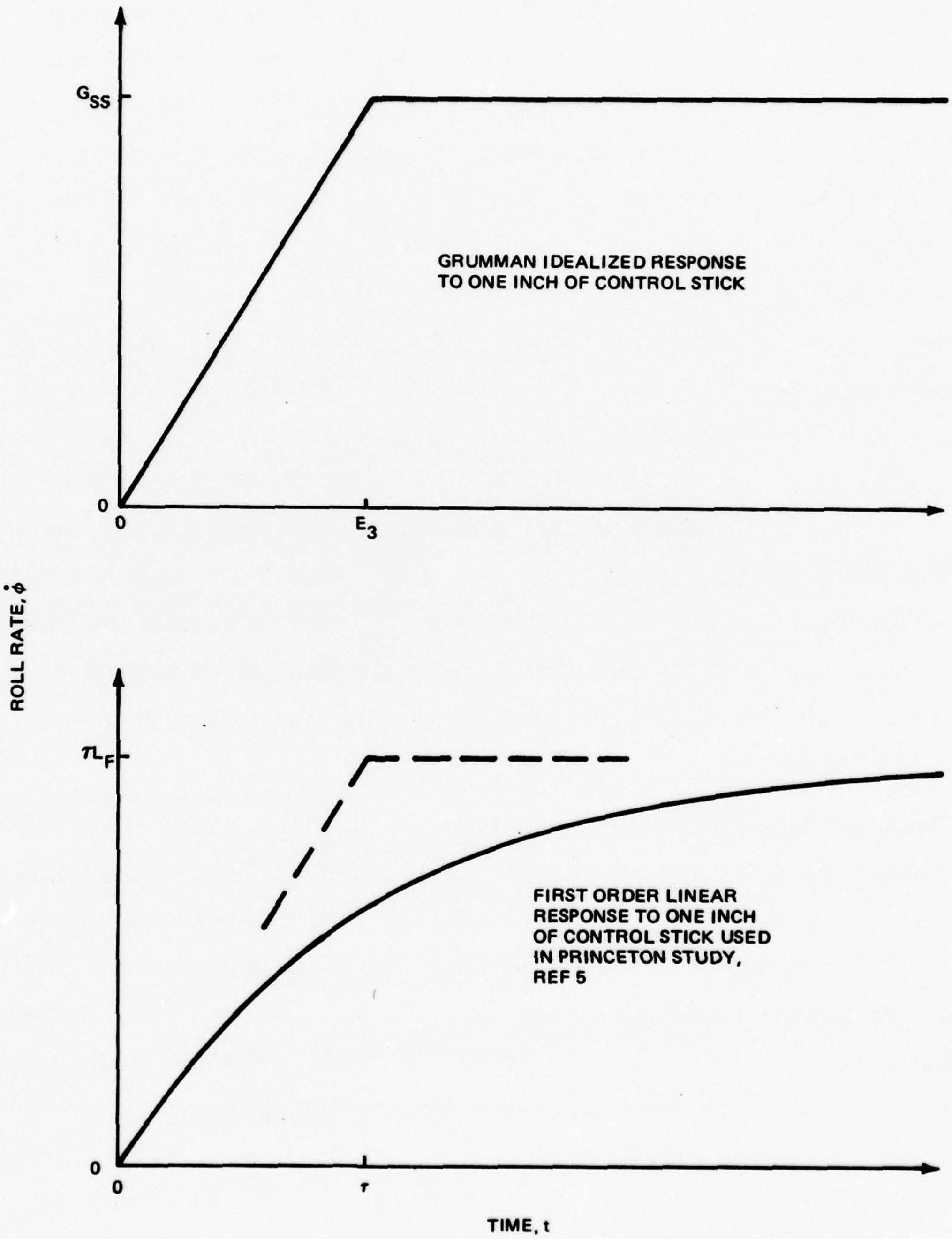


Fig. 13 Comparison of First Order Linear Roll Rate Response and the Grumman Idealized Roll Rate Response

studies. Results of both studies are shown in Figure 14. The Grumman data is a transformation of the 95 percent region of Figure 12 onto the plane of Figure 14, while the Princeton data is a reproduction of the way it was presented in Ref. 5. Although the Princeton results show a slightly wider range of satisfactory τ's than the comparable range of E_3's for the 95 percent region, the results are really quite similar -- especially in light of our uncertainty about what level of pilot preference (95 percent, 90 percent, 85 percent,...) is comparable to a "satisfactory" rating of 3 on the Cooper Scale (Ref. 1). A slightly lower level of pilot preference would have a range of "rise-times" almost identical to Princeton's satisfactory region.

The principal discrepancy between the two sets of data is the difference in the general level of "initial acceleration" found to be satisfactory. Here we feel that this difference can be largely explained by the one major difference between the two experimental set-ups: stick force gradient. This parameter was not varied in either study; consequently, we could not assess its direct effect on these data. The stick force gradient is generally known to have a powerful effect on flying qualities in tracking tasks (see, for instance, Ref. 6), and there was a big difference between the values used in the two studies. For the Princeton study the stick force gradient was 4.5 pounds/inch (representative of contemporary jet fighter aircraft), while for our work it was only 0.5 pound/inch (more representative of hovering VTOL craft). The much higher value used in the Princeton study would make the pilots want a higher "initial acceleration" gain to reduce the physical effort

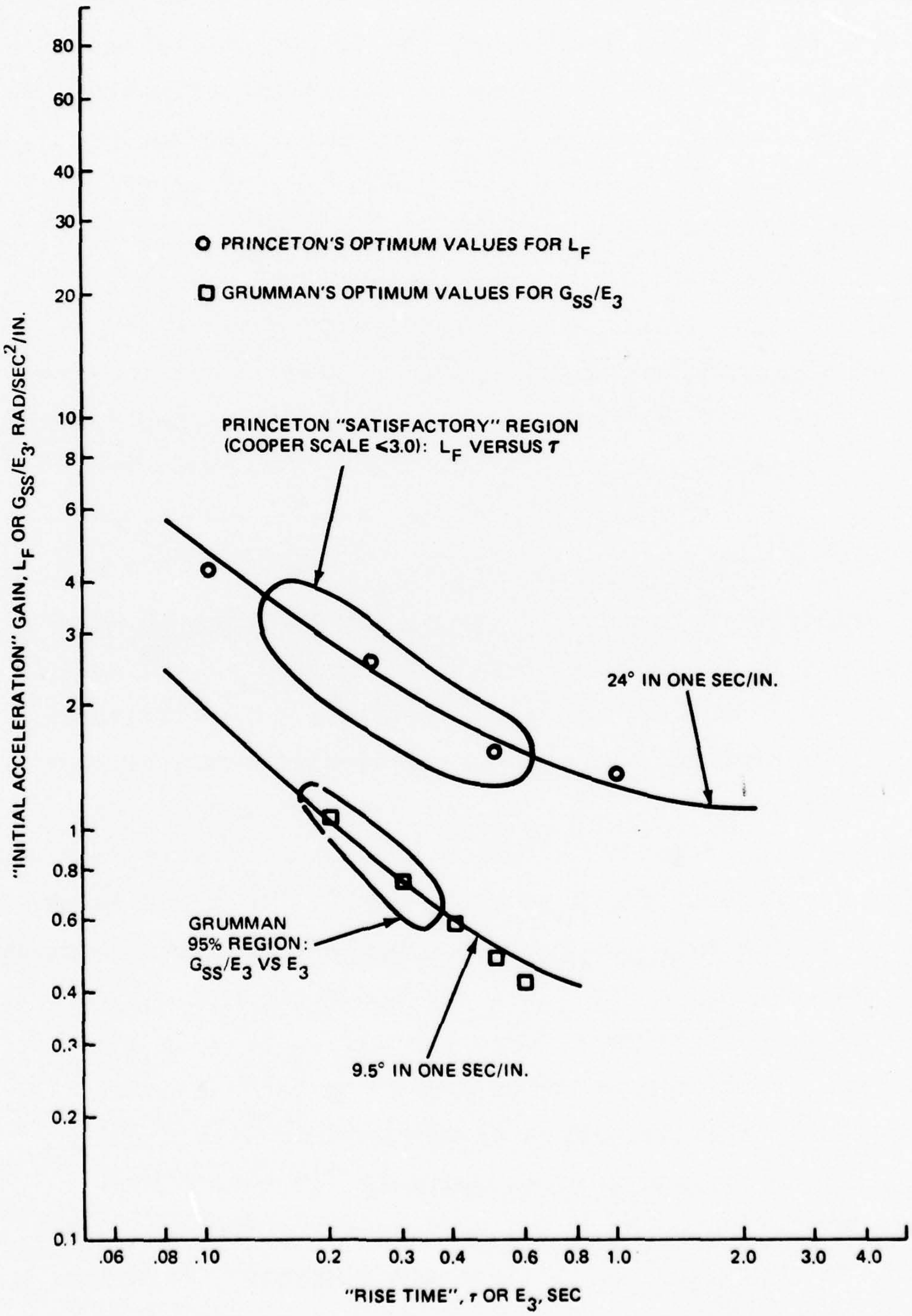
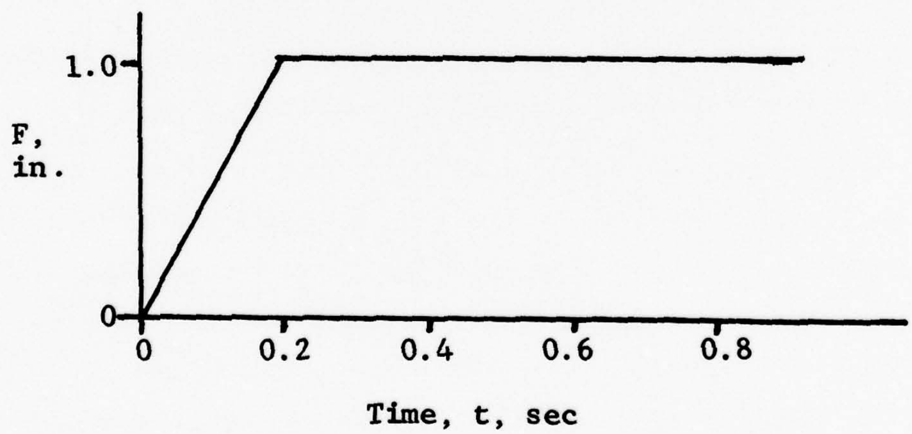


Fig. 14 Comparison of Grumman Data with Princeton University Data (Ref. 5)

required to manipulate the controller. Conversely, the much lower value used in our study would make the pilots want a lower "initial acceleration" gain to reduce any tendency toward over-control and pilot-induced-oscillation (PIO). With this explanation in mind, and remembering that the step responses are not exactly alike in the two studies, we conclude that our results are wholly reasonable in comparison to the Princeton flight test data. The Princeton report also showed that the optimum configurations could be expressed in terms of a given bank angle attained in one second for one inch stick deflection. It turns out that the optimum region is about the same for all roll time-constants and corresponds to a 24 degree bank angle in one second (see Fig. 14) for the control input shown below.



For comparison, we tried to develop something similar for our 95 percent region using the same control input. Figure 14 shows that a line of 9.5 degrees in one second for one inch control deflection closely matches our optimum region. This hints at the possibility that pilots regard their ability to achieve a given bank angle within one second for a given control effort as a fundamental measure of a system's worth in a roll tracking task. We saw in Figures 9 and 12, however, that steady

state gain, G_{ss}, might be a fundamental parameter that pilots choose independently of E_3. Thus our data suggests a hypothesis that says, "the pilot always chooses the same steady state gain, G_{ss}, (for a given stick force gradient) independent of the transient character of the response," but when viewed differently it also suggests, along with the Princeton data, a hypothesis that says, "the pilot chooses, G_{ss}, such that, with a certain step-like command of one inch, the aircraft will always achieve a specified bank angle in one second (again for a given stick force gradient)." Clearly, both cannot be true, and the data gathered so far do not allow us to determine which (if indeed either) is really correct. This is pointed out to demonstrate how elusive basic flying qualities parameters can be, and that the convolution technique, with its ability to simulate systems easily with unconventional or even strange responses, can be a valuable tool for testing hypothesis such as these.

Our results are somewhat elementary, but unlike conventional handling qualities data they could form a basis for future study of roll control of highly augmented craft, where nonclassical responses may occur and possibly be preferred. (The regions of relative pilot preference would, of course, first have to be redefined by "expert" pilots into regions of absolute pilot opinion, most likely using the Cooper-Harper pilot opinion scale.) We could, for instance, begin to explore how the most preferred regions are affected by variations from the idealized response such as "rounding" of corners (i.e., nonzero values of E_2 and E_4 in Fig. 2), oscillatory overshoots, initial control reversals, and various other control system anomalies. This could be done without regard to the complicated dynamics of the aircraft and control system except as depicted by the kinematics of the response itself.

Grand Experiment 2 was designed as a tentative beginning to such work and attempted to determine the relative effects of two elementary kinds of response degradations that could result from a variety of airframe/control-system characteristics. The variables were the E_1 and E_2 elements of the roll rate step response shown in Figure 2. E_1 is a pure time delay, and E_2 can be thought of as a "lagged" time delay or a time delay with a rounded corner similar to that which is achieved by some conventional lagged responses. We refer to the sum of E_1 and E_2 $(E_1 + E_2)$ as the "total effective" time delay. For these tests E_3 was 0.4 second and G_{ss} was 12 deg/sec/in. This puts the base configuration ($E_1 = E_2 = 0$) within the 90 percent region of pilot preference (Fig. 12), but with an E_3 about 0.1 second greater than the optimum value. E_4 was once again zero.

Figure 15 is a three-dimensional presentation of the data which shows the gross effects quite nicely. Note that the "best" and "worst" designations of this figure refer to the configurations of Grand Experiment 2 only, and that the pilot preference scale used here is different from the scale used for the results of Grand Experiment 1 (Figures 9, 10 and 11). For instance, remember that the best configuration here (i.e., $E_1 = E_2 = 0$, $E_3 = 0.4$ second and $E_4 = 12$ deg/sec/in.) is only 90 to 95 percent as good as the best configuration in Grand Experiment 1 (see Figure 12). The most obvious result is that increasing either E_1 or E_2 always degraded pilot preference. This is not very surprising, and, in fact, merely justifies the assumption that was made to that effect during the experimental design to allow a reduction in the size of the experiment (see page 39). The effects of both E_1 and E_2 are quite strong as values of about 0.1 second for either

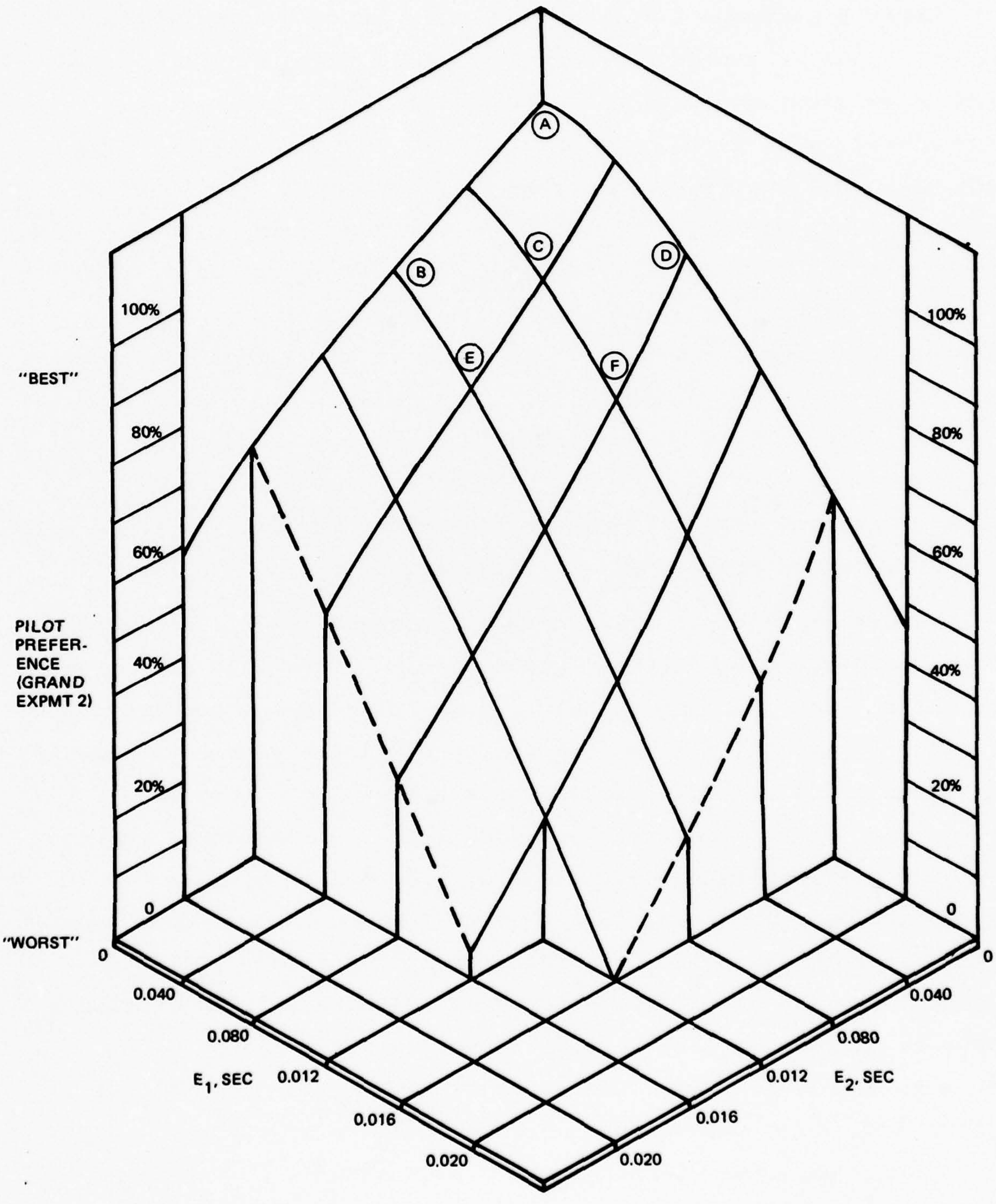


Fig. 15 Summary of Results of Grand Experiment 2: Pilot Preference versus E_1 and E_2

one cause almost a 20 percent reduction in pilot preference. This is regarded as substantial because of the pilot's report that only the very best configurations were really any good.

We are really much more interested in the relative effects of E_1 and E_2, and these are difficult to discern from Figure 15. To help clarify these effects, Figure 16 has been prepared to show how pilot preference of several representative E_1, E_2 configurations changes when either E_1 or E_2 is increased from its starting value. The alphabetical labels indicate where the starting E_1, E_2 configurations are located on Figure 15, and the shaded areas surrounding each curve portray the standard deviation of the data. The results are not definitive. For configurations D and F there appears to be no measurable difference between increasing E_1 or E_2, as the two curves never separate beyond the standard deviation of the data. For the remaining four configurations A, B, C and E, however, there does appear to be a small but measurable difference for sufficiently large increases in E_1 and E_2. We conclude that E_1 is probably more detrimental than E_2, but the difference between the two effects is small, difficult to measure, and generally not detectable for increases less than about 0.10 second. This does not necessarily mean that the pilots couldn't sense smaller differences, but rather that the smaller differences (sensed or not) provided no perceptible advantage or disadvantage in performance of the tracking task.

Figure 17 shows the step responses for four pairs of configurations that lie on the first set of curves in Figure 16 at the points labeled 1, 2, 3, and 4, respectively. That is, the step response shown on the left in pair 4 of Figure 17 has the pilot preference indicated by the upper curve of Figure 16a

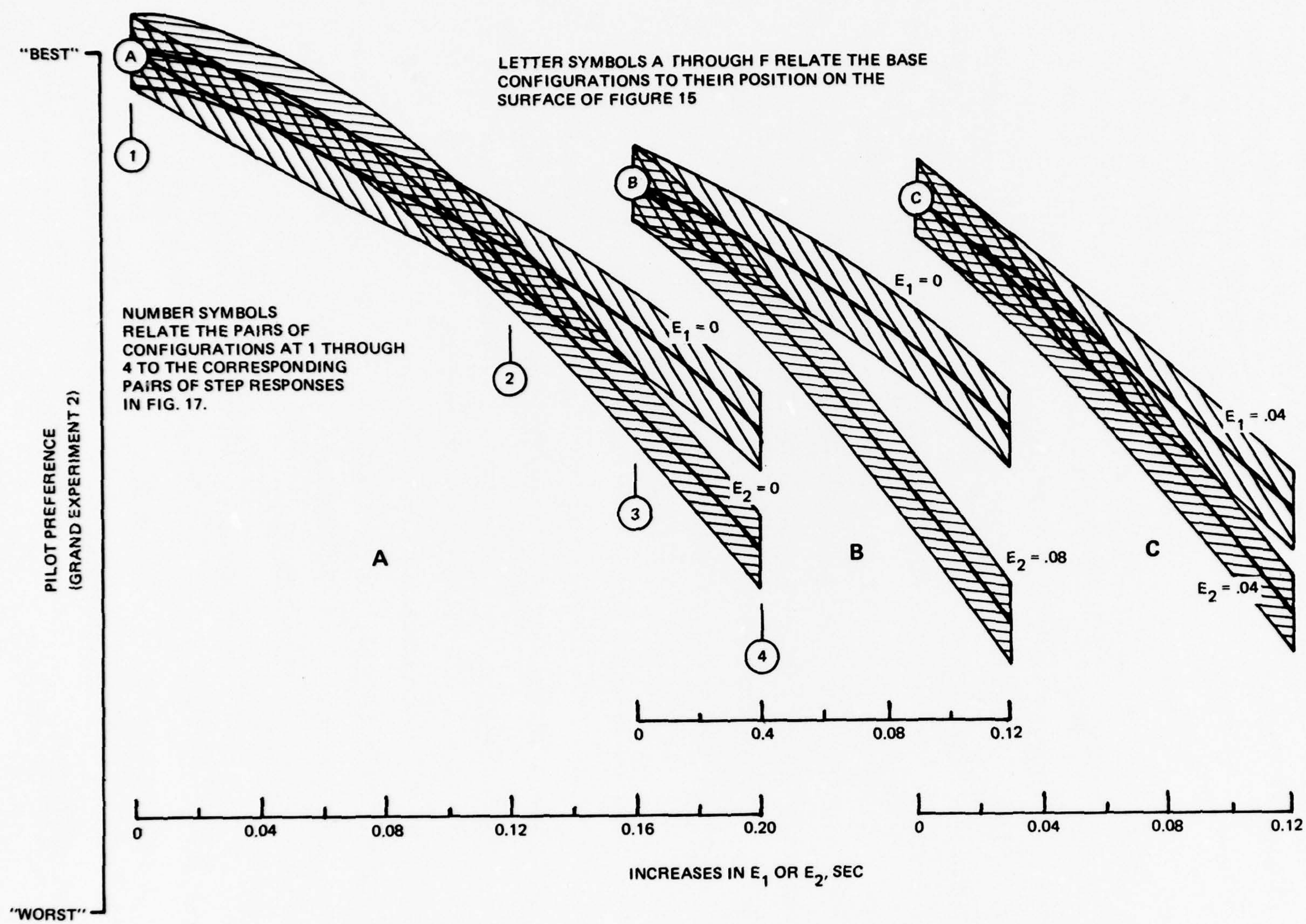


Fig. 16 Effects of E_1 and E_2 on Several Representative Configurations, Sheet 1 of 2

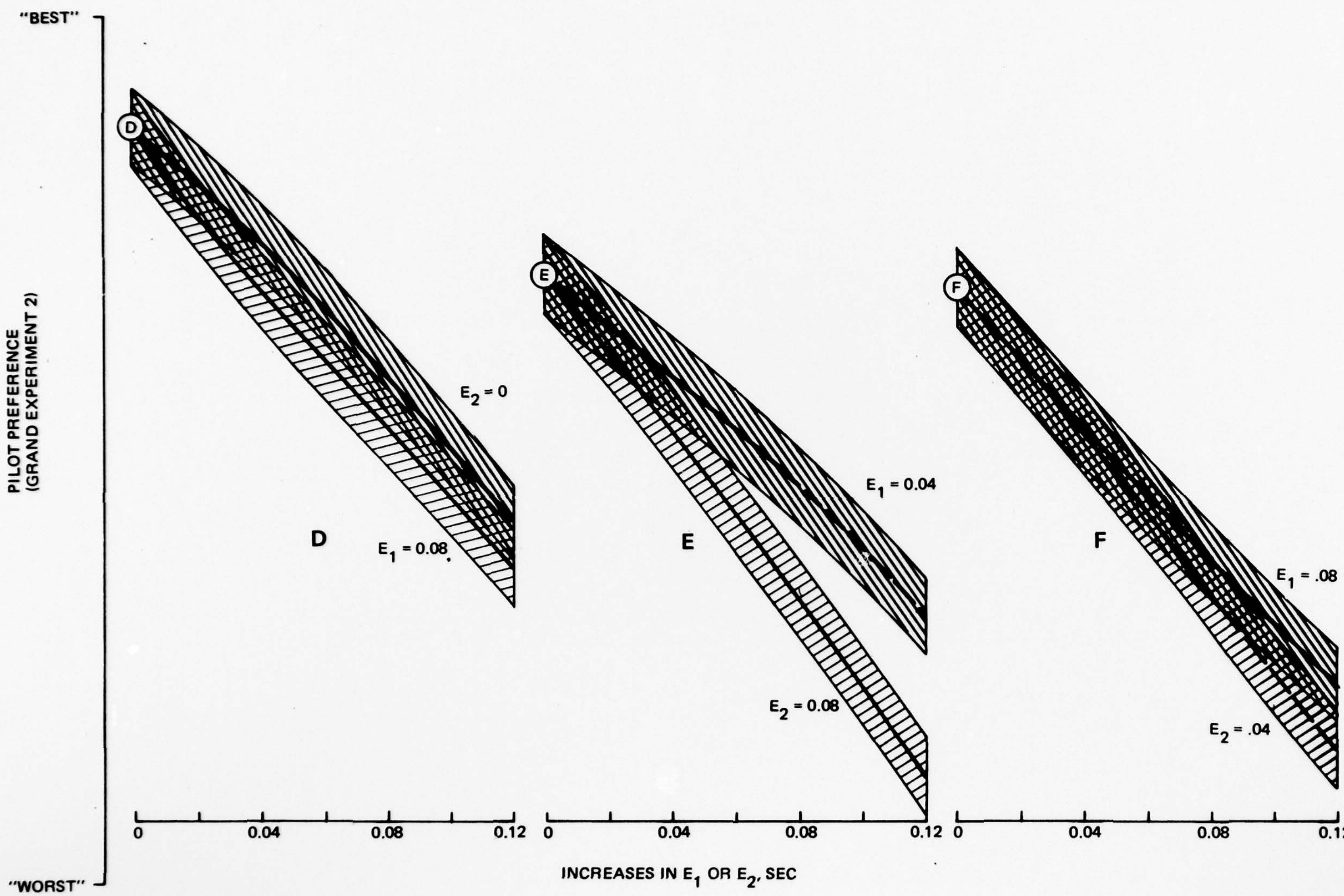


Fig. 16 Effects of E_1 and E_2 on Several Representative Configurations, Sheet 2 of 2

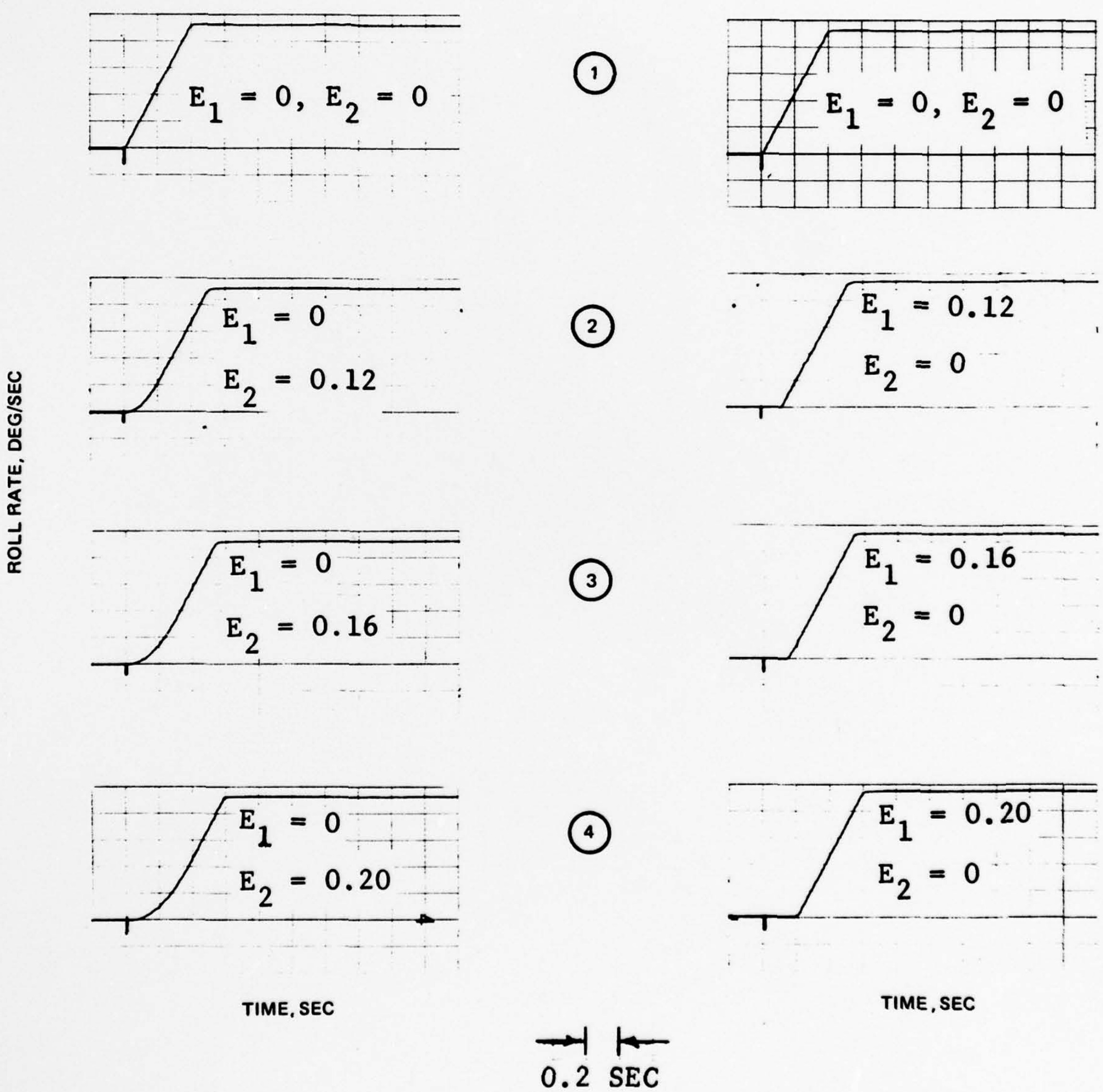


Fig. 17 Pairs of Step Response Time Histories for the Configurations Called-Out on the First Pair of Curves on Figure 16

at 4. The curves shown in Figure 17 are actual strip chart recordings of time histories produced by our real time convolution routine in response to step inputs. They portray the differences in vehicle step responses that are caused by increasing E_1 (left hand column of responses) versus increasing E_2 (right hand column of responses), and in association with Figure 16 they provide a feeling for the magnitude of these differences needed to produce measurable differences in pilot preference.

5. CONCLUSIONS

A novel real-time simulation technique based on the mathematical concept of convolution has been mechanized on a digital computer to drive a single degree of freedom roll simulator, and exploratory roll-control handling-qualities studies have been performed. The major thrust of this effort was to demonstrate that the convolution simulation technique allows researchers to study flying qualities in a different way -- a way that has particular significance to the next generation of military aircraft that will have radical improvements in maneuverability due to the addition of new types of control surfaces and the implementation of digital fly-by-wire control schemes.

The flying qualities results discussed in Section 4 are obviously rudimentary, and many more experiments are necessary to define fully the preferred roll rate response, even in the simple context used here. The experimentation did show, however, that direct manipulation of the kinematics of the pilot's cockpit for the purpose of defining handling qualities requirements is feasible. The potential practicality of this technique stems from the ever increasing complexity of the analytic descriptors of the aircraft and control system responses and the concurrent need to define flying qualities as simply as possible. Researchers have often sought to write flying qualities specifications as time domain requirements (see Refs. 7 and 8), but have always done their experimentation with other than time domain parameters. We believe that this is becoming more and more impractical, and that the technique proposed here can be a very useful aid in studying flying qualities requirements of

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highly augmented craft. Thus we think that a long-range program of intensive experimentation should now follow, one that uses this new approach to uncover the control responses that pilots most prefer. Then we will be able to use the power of modern control system technology most effectively and provide the best response at each flight condition.

A particularly rich and timely application is the study of hover control of VTOL craft. The next generation of these craft should finally achieve operational status with the military, where success will depend heavily on their flying qualities at hover. Current thinking has them employing uncoupled, highly augmented fly-by-wire schemes that will produce nonclassical control responses at this crucial flight condition. This problem is particularly amenable to study with the convolution simulation technique, and we urge that it be so addressed.

6. REFERENCES

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APPENDIX

DATA AND ANALYSIS

The data from experiments 1.1 through 1.8 (composing Grand Experiment 1) are presented in the proportion matrices and scale separation matrices of Tables A-1 through A-8. Also, the data from experiments 2.1 through 2.8 (composing Grand Experiment 2) are presented in the proportion and scale separation matrices of Tables A-9 through A-16. All the matrices are arranged so that the configurations heading the columns are in increasing order of rank of their final scale values. This was achieved before the actual development of the scale by ordering them according to the average value of the proportion matrix columns. In each table the scale separation matrix is derived from the data in the proportion matrix using normal curve tables and Eq. (7) (see Section 3). The final scale values (R_j) were then calculated at the bottom of each scale separation matrix (ΔR_{jk}).

TEST OF INTERNAL CONSISTENCY

We will demonstrate the test of internal consistency originally proposed by Mosteller (Ref. 4) and later demonstrated by Guilford (Ref. 3) on the data of experiment 1.1 shown in Table A-1. It amounts to making a chi-square test of the significance between the P_{jk} matrix and a matrix of proportions that would be expected, P'_{jk}, given the scale values at the bottom of the scale separation matrix. Clearly we wish the test to be failed and thus indicate that there is only an insignificant amount of chance inconsistency between the raw data and the scale produced. We start by using the R_j scale at the bottom of the ΔR_{jk} matrix (Table A-1) to calculate the elements of the expected scale separation matrix $\Delta R'_{jk}$ in Table A-17. Next we again use normal curve tables and Eq. (7), but this time we transform the $\Delta R'_{jk}$ matrix into the P'_{jk} matrix also shown in

Table A-17. The P_{jk} matrix of Table A-1 must now be compared for consistency with the P'_{jk} matrix of Table A-17. Equation (A-1) is the formula for chi-square that applies to this situation (Ref. 4).

$$\chi^2 = \frac{N}{821} \sum (\theta - \theta')^2 \qquad \text{(A-1)}$$

where:

N = number of times each pair of stimuli are judged

the statistic $\theta = \text{Sin}^{-1} \sqrt{P_{jk}}$

and $\theta' = \text{Sin}^{-1} \sqrt{P'_{jk}}$

For the Table A-1 data, N = 20 and chi-square is calculated to be 4.91. Reference 4 gives the formula for the number of degrees of freedom for this situation as

$$df = \frac{(n-1)(n-2)}{2}$$

Where: n = number of configurations.

Again for the case being studied here n = 5 and df = 6. Reference to a table of chi-square shows that it takes a chi-square as large as 12.59 to be significant at the 0.05 level. We may say that the obtained chi-square (4.91) is insignificant.

The conclusion to be drawn from such a result is that the scale produced is consistent with the data, and that the assumptions made to produce the scale (see page 21) have not introduced a significant amount of error.

CURVE FITTING

We recall that the pilot preference results developed from each pair comparison experiment are on an interval scale. Thus

we are allowed to adjust all the pilot preferences from a single experiment by a constant amount without destroying the essential character of the results: the relative preference of configurations. Also, there are many configurations in each grand experiment that are evaluated in two separate experiments. For instance, C_{13} is at the intersection of experiments 1.3 and 1.7 (Fig. 6) and is given a scale value of 1.185 from the data of one (Table A-3) and another value, 1.749, from the data of the other (Table A-7). We would like to reconcile these differences and plot the results of all experiments composing a grand experiment against the same scale of pilot preference. (This is allowed if we assume that the term in brackets in Eq. (6) in Section 3 is constant over a grand experiment.) We do this by selecting a set of constants (one for each scale) that will minimize the sum of squares of all differences between scale values produced by two experiments for the same configuration in a grand experiment (there are 15 such differences in Grand Experiment 1 and 16 in Grand Experiment 2, as shown in Figures 6 and 7, respectively). To improve the numerics of the least squares solution process we arbitrarily picked a constant of 10 for one of the scales from each grand experiment. This had the effect of changing the range of values from 0 to 3.0 for the basic scales to 10 to 13 for the adjusted scales. The basic scales produced by each experiment are summarized (from Table A-1 through A-16) in Tables A-18 and A-19 and the scales adjusted to minimize the sum of squares of the intersect errors are summarized in Tables A-20 and A-21.

Next we made a least squares fit of a third order polynomial surface to the adjusted pilot preference data of each grand experiment. The resultant polynomials for Grand Experiments 1

and 2 are as follows.

$$\begin{aligned}\text{Pilot Preference (Grand Experiment 1)} &= 6.8416 + 2.8026\ E_3 + .9774\ G_{ss} \\ &- 6.7805\ E_3^{\ 2} + .0886\ E_3\ G_{ss} - .0583\ G_{ss}^{\ 2} \\ &+ .5644\ E_3^{\ 3} - .3632\ E_3^{\ 2}\ G_{ss} + .0144\ E_3 G_{ss}^{\ 2} \\ &+ .00085\ G_{ss}^{\ 2} \qquad \text{(A-2)}\end{aligned}$$

$$\begin{aligned}\text{Pilot Preference (Grand Experiment 2)} &= 12.742 + .248\ E_1 - 6.271\ E_2 - 65.468\ E_1^{\ 2} \\ &- 114.256\ E_1 E_2 + 19.482\ E_2^{\ 2} + 122.332 E_1^{\ 3} \\ &+ 223.923 E_1^{\ 2} E_2 + 166.619\ E_1 E_2^{\ 2} \\ &- 92.999\ E_2^{\ 3} \qquad \text{(A-3)}\end{aligned}$$

The goodness of fit of these polynomials to their respective data bases is indicated by the standard deviation of the data from the polynomial surface, σ, and by the correlation coefficient, ρ. For the polynomial fit to the data of Grand Experiment 1 (Eq. A-2),

$$\sigma = 0.1632$$

and $$\rho = 0.9684$$

Likewise for thepolynomial fit to the data of Grand Experiment 2 (Eq. A-3),

$$\sigma = 0.1218$$

and $$\rho = 0.9870$$

A more graphic portrayal of the goodness of fit is given by Figures A-1 and A-2. These show many planar sections of the polynomial surfaces and the relationship of adjusted scale value data to the curves. By all standards considered the polynomial surfaces of Eq. (A-2) and (A-3) are good representations of the

experimental results. Thus, they are considered to be the final fairing of the data, and were used to generate the pilot preference curves discussed in the main body of the report.

TABLE A-1 THE PROPORTION MATRIX (P_{jk}) AND SCALE SEPARATION MATRIX (ΔR_{jk}) FROM EXPERIMENT 1.1

P_{jk} = (N=20)

C_k \ C_j	C_5	C_1	C_4	C_2	C_3
C_5	.500	.650	.900	.900	1.000
C_1	.350	.500	.500	.900	.850
C_4	.100	.500	.500	.650	.900
C_2	.100	.100	.350	.500	.750
C_3	.0	.150	.100	.250	.500

ΔR_{jk} =

C_k \ C_j	C_5	C_1	C_4	C_2	C_3
C_5	0	.3853	1.2816	1.2816	---
C_1	- .3853	0	0	1.2816	1.0364
C_4	-1.2816	0	0	.3853	1.2816
C_2	-1.2816	-1.2816	- .3853	0	.6745
C_3	---	-1.0364	-1.2816	- .6745	0
ΔR_j	---	.513	.309	.532	.500
R_j	0	.513	.822	1.354	1.854

Because there are elements of ΔR_{jk} missing, the ΔR_j values are computed as follows:

$$\Delta R_j = \sum^{k} (R_{jk} - R_{j-1,k})/\text{no. of k's}$$

- Chi square for the test of internal consistency: $\chi^2 = 4.91$
- For R_j to be significantly inconsistent with the data at the 0.05 level requires a chi square of 12.59

TABLE A-2 THE PROPORTION MATRIX (P_{jk}) AND SCALE SEPARATION MATRIX (ΔR_{jk}) FROM EXPERIMENT 1.2

P_{jk} = (N=20)

C_k \ C_j	C_6	C_{10}	C_7	C_9	C_8
C_6	.500	.65	.75	.65	.90
C_{10}	.35	.500	.90	1.00	.80
C_7	.25	.100	.500	.65	.70
C_9	.35	0	.35	.500	.65
C_8	.10	.20	.30	.35	.500

ΔR_{jk} =

C_k \ C_j	C_6	C_{10}	C_7	C_9	C_8
C_6	0	.3853	.6745	.3853	1.2816
C_{10}	-.3853	0	1.2816	---	.8416
C_7	-.6745	-1.2816	0	.3853	.5244
C_9	-.3853	---	-.3853	0	.3853
C_8	-1.2816	-.8416	-.5244	-.3853	0
ΔR_j	---	.151	.792	.155	.452
R_j	0	.151	.943	1.098	1.55

Because there are elements of ΔR_{jk} missing, the ΔR_j values are computed as follows:

$$\Delta R_j = \sum^{k} (R_{jk} - R_{j-1,k})/\text{no. of } k\text{'s}$$

- Chi square for the test of internal consistency: $\chi^2 = 4.97$
- For R_j to be significantly inconsistent with the data at the 0.05 level requires a chi square of 12.59

TABLE A-3 THE PROPORTION MATRIX (P_{jk}) AND SCALE SEPARATION MATRIX (ΔR_{jk}) FROM EXPERIMENT 1.3

P_{jk} = (N=20)

C_k \ C_j	C_{11}	C_{15}	C_{12}	C_{14}	C_{13}
C_{11}	.500	.450	.900	.850	1.000
C_{15}	.550	.500	.650	.650	.650
C_{12}	.100	.350	.500	.550	.800
C_{14}	.150	.350	.450	.500	.650
C_{13}	0	.350	.200	.350	.500

ΔR_{jk} =

C_k \ C_j	C_{11}	C_{15}	C_{12}	C_{14}	C_{13}
C_{11}	0	- .1257	1.2816	1.0364	---
C_{15}	+ .1257	0	.3853	.3853	.3853
C_{12}	-1.2816	- .3853	0	.1257	.8416
C_{14}	-1.0364	- .3853	- .1257	0	.3853
C_{13}	---	- .3853	- .8416	- .3853	0
ΔR_j	---	.324	.396	.093	.372
R_j	0	.324	.720	.813	1.185

Because there are elements of ΔR_{jk} missing, the ΔR_j values are computed as follows:

$$\Delta R_j = \sum^{k} (R_{jk} - R_{j-1,k})/\text{no. of } k\text{'s}$$

- Chi square for the test of internal consistency: $\chi^2 = 9.47$
- For R_j to be significantly inconsistent with the data at the 0.05 level requires a chi square of 12.59

TABLE A-4 THE PROPORTION MATRIX (P_{jk}) AND SCALE SEPARATION MATRIX (ΔR_{jk}) FROM EXPERIMENT 1.4

P_{jk} =

(N=20)

C_k \ C_j	C_{16}	C_{20}	C_{17}	C_{19}	C_{18}
C_{16}	.500	.600	.900	.850	.800
C_{20}	.400	.500	.550	.650	.600
C_{17}	.100	.450	.500	.600	.800
C_{19}	.150	.350	.400	.500	.500
C_{18}	.200	.400	.200	.500	.500

ΔR_{jk} =

C_k \ C_j	C_{16}	C_{20}	C_{17}	C_{19}	C_{18}
C_{16}	0	.2533	1.2816	1.0364	.8416
C_{20}	- .2533	0	.1257	.3853	.2533
C_{17}	-1.2816	- .1257	0	.2533	.8416
C_{19}	-1.0364	- .3853	- .2533	0	0
C_{18}	- .8416	- .2533	- .8416	0	0
ΔR_j	---	.580	.165	.273	.052
R_j	0	.580	.745	1.018	1.070

- Chi square for the test of internal consistency: $\chi^2 = 6.35$
- For R_j to be significantly inconsistent with the data at the 0.05 level requires a chi square of 12.59

TABLE A-5 THE PROPORTION MATRIX (P_{jk}) AND SCALE SEPARATION MATRIX (ΔR_{jk}) FROM EXPERIMENT 1.5

P_{jk} = (N=20)

C_k \ C_j	C_{21}	C_{22}	C_{25}	C_{23}	C_{24}
C_{21}	.500	.850	.700	.950	.900
C_{22}	.150	.500	.400	.950	.850
C_{25}	.300	.600	.500	.600	.750
C_{23}	.050	.050	.400	.500	.650
C_{24}	.100	.150	.250	.350	.500

ΔR_{jk} =

C_k \ C_j	C_{21}	C_{22}	C_{25}	C_{23}	C_{24}
C_{21}	0	1.0364	.5244	1.6449	1.2816
C_{22}	-1.0364	0	.2533	1.6449	1.0364
C_{25}	- .5244	- .2533	0	.2533	.6745
C_{23}	-1.6449	-1.6449	- .2533	0	.3853
C_{24}	-1.2816	-1.0364	- .6745	- .3853	0
ΔR_j	---	.619	.147	.763	.044
R_j	0	.619	.766	1.529	1.573

- Chi square for the test of internal consistency: $\chi^2 = 11.04$
- For R_j to be significantly inconsistent with the data at the 0.05 level requires a chi square of 12.59

TABLE A-6 THE PROPORTION MATRIX (P_{jk}) AND SCALE SEPARATION MATRIX (ΔR_{jk}) FROM EXPERIMENT 1.6

P_{jk} =

(N=20)

C_k \ C_j	C_{21}	C_{16}	C_{11}	C_6	C_1
C_{21}	.500	.550	.700	1.000	.900
C_{16}	.450	.500	.650	.700	.950
C_{11}	.300	.350	.500	.650	.550
C_6	0	.300	.350	.500	.500
C_1	.100	.050	.450	.500	.500

ΔR_{jk} =

C_k \ C_j	C_{21}	C_{16}	C_{11}	C_6	C_1
C_{21}	0	.1257	.5244	---	1.2816
C_{16}	- .1257	0	.3853	.5244	1.6449
C_{11}	- .5244	- .3853	0	.3853	.1257
C_6	---	- .5244	- .3853	0	0
C_1	-1.2816	-1.6449	- .1257	0	0
ΔR_j	---	.007	.566	.259	.215
R_j	0	.007	.573	.832	1.047

Because there are elements of ΔR_{jk} missing, the ΔR_j values are computed as follows:

$$\Delta R_j = \sum^{k} (R_{jk} - R_{j-1,k})/\text{no. of k's}$$

- Chi square for the test of internal consistency: $\chi^2 = 4.34$
- For R_j to be significantly inconsistent with the data at the 0.05 level requires a chi square of 12.59

TABLE A- 7 THE PROPORTION MATRIX (P_{jk}) AND SCALE SEPARATION MATRIX (ΔR_{jk}) FROM EXPERIMENT 1.7

P_{jk} = (N=40)

C_k \ C_j	C_{23}	C_{18}	C_3	C_8	C_{13}
C_{23}	.500	.850	.900	.850	.925
C_{18}	.150	.500	.675	.750	.625
C_3	.100	.325	.500	.550	.625
C_8	.150	.250	.450	.500	.550
C_{13}	.075	.375	.375	.450	.500

ΔR_{jk} =

C_k \ C_j	C_{23}	C_{18}	C_3	C_8	C_{13}
C_{23}	0	1.0364	1.2816	1.0364	1.4395
C_{18}	-1.0364	0	.4538	.6745	.3186
C_3	-1.2816	- .4538	0	.1257	.3186
C_8	-1.0364	- .6745	- .1257	0	.1257
C_{13}	-1.4395	- .3186	- .3186	- .1257	0
ΔR_j	---	1.0959	.4254	.1049	.1229
R_j	0	1.0959	1.5213	1.6262	1.7491

- Chi square for the test of internal consistency: $\chi^2 = 9.47$
- For R_j to be significantly inconsistent with the data at the 0.05 level requires a chi square of 12.59

TABLE A-8 THE PROPORTION MATRIX (P_{jk}) AND SCALE SEPARATION MATRIX (ΔR_{jk}) FROM EXPERIMENT 1.8

P_{jk} =

(N=20)

C_k \ C_j	C_5	C_{10}	C_{25}	C_{15}	C_{20}
C_5	.500	.750	.700	.800	.650
C_{10}	.250	.500	.550	.550	.800
C_{25}	.300	.450	.500	.600	.600
C_{15}	.200	.450	.400	.500	.600
C_{20}	.350	.200	.400	.400	.500

ΔR_{jk} =

C_k \ C_j	C_5	C_{10}	C_{25}	C_{15}	C_{20}
C_5	0	.6745	.5244	.8416	.3853
C_{10}	-.6745	0	.1257	.1257	.8416
C_{25}	-.5244	-.1257	0	.2533	.2533
C_{15}	-.8416	-.1257	-.2533	0	.2533
C_{20}	-.3853	-.8416	-.2533	-.2533	0
ΔR_j	---	.401	.112	.165	.153
R_j	0	.401	.513	.678	.831

- Chi square for the test of internal consistency: $\chi^2 = 5.01$
- For R_j to be significantly inconsistent with the data at the 0.05 level requires a chi square of 12.59

TABLE A-9 THE PROPORTION MATRIX (P_{jk}) AND SCALE SEPARATION MATRIX (ΔR_{jk}) FROM EXPERIMENT 2.1

P_{jk} = (N=20)

C_k \ C_j	C_{47}	C_{46}	C_{41}	C_{36}	C_{31}	C_{13}
C_{47}	.500	.80	.70	.90	--	--
C_{46}	.20	.500	.65	.90	--	--
C_{41}	.30	.35	.500	.80	.65	.80
C_{36}	.10	.10	.20	.500	.65	.60
C_{31}	--	--	.35	.35	.500	.50
C_{13}	--	--	.20	.40	.50	.500

ΔR_{jk} =

C_k \ C_j	C_{47}	C_{46}	C_{41}	C_{36}	C_{31}	C_{13}
C_{47}	0	.8416	.5244	1.2816	---	---
C_{46}	- .8416	0	.3853	1.2816	---	---
C_{41}	- .5244	- .3853	0	.8416	.3853	.8416
C_{36}	-1.2816	-1.2816	- .8416	0	.3853	.2533
C_{31}	---	---	- .3853	- .3853	0	0
C_{13}	---	---	- .8416	- .2533	0	0
ΔR_j	---	.4566	.2234	.6542	.1419	.0811
R_j	0	.4566	.6790	1.3332	1.4751	1.5562

Because there are elements of ΔR_{jk} missing, the ΔR_j values are computed as follows:

$$\Delta R_j = \sum^{k} (R_{jk} - R_{j-1,k})/\text{no. of k's}$$

- Chi square for the test of internal consistency: $\chi^2 = 9.04$
- For R_j to be significantly inconsistent with the data at the 0.05 level requires a chi square of 18.31

TABLE A-10 THE PROPORTION MATRIX (P_{jk}) AND SCALE SEPARATION MATRIX (ΔR_{jk}) FROM EXPERIMENT 2.2

P_{jk} =

C_k \ C_j	C_{48}	C_{42}	C_{37}	C_{32}	C_{26}
C_{48}	.500	.90	.75	.90	---
C_{42}	.10	.500	.60	.75	.85
C_{37}	.25	.40	.500	.60	.80
C_{32}	.10	.25	.40	.500	.50
C_{26}	---	.15	.20	.50	.500

(N=20)

ΔR_{jk} =

C_k \ C_j	C_{48}	C_{42}	C_{37}	C_{32}	C_{26}
C_{48}	0	1.2816	.6745	1.2816	---
C_{42}	-1.2816	0	.2533	.6745	1.0364
C_{37}	- .6745	- .2533	0	.2533	.8416
C_{32}	-1.2816	- .6745	- .2533	0	0
C_{26}	---	-1.0364	- .8416	0	0
ΔR_j	---	.8979	.1031	.4753	.2375
R_j	0	.8979	1.0010	1.4763	1.7138

Because there are elements of ΔR_{jk} missing, the ΔR_j values are computed as follows:

$$\Delta R_j = \sum^{k} (R_{jk} - R_{j-1,k})/\text{no. of k's}$$

- Chi square for the test of internal consistency: $\chi^2 = 5.32$
- For R_j to be significantly inconsistent with the data at the 0.05 level requires a chi square of 12.59

TABLE A-11 THE PROPORTION MATRIX (P_{jk}) AND SCALE SEPARATION MATRIX (ΔR_{jk}) FROM EXPERIMENT 2.3

P_{jk} = (N=40)

C_k \ C_j	C_{49}	C_{43}	C_{38}	C_{33}	C_{27}
C_{49}	.500	.700	.925	.900	---
C_{43}	.300	.500	.800	.825	.975
C_{38}	.075	.200	.500	.675	.800
C_{33}	.100	.175	.325	.500	.800
C_{27}	---	.025	.200	.200	.500

ΔR_{jk} =

C_k \ C_j	C_{49}	C_{43}	C_{38}	C_{33}	C_{27}
C_{49}	0	.5244	1.4395	1.2816	---
C_{43}	-.5244	0	.8416	.9346	1.9600
C_{38}	-1.4395	-.8416	0	.4538	.8416
C_{33}	-1.2816	-.9346	-.4538	0	.8416
C_{27}	---	-1.9600	-.8416	-.8416	0
ΔR_j	---	.4984	.8396	.1684	.7741
R_j	0	.4984	1.3380	1.5064	2.2805

Because there are elements of ΔR_{jk} missing, the ΔR_j values are computed as follows:

$$\Delta R_j = \sum^{k} (R_{jk} - R_{j-1,k}) / \text{no. of } k\text{'s}$$

- Chi square for the test of internal consistency: $\chi^2 = 3.67$
- For R_j to be significantly inconsistent with the data at the 0.05 level requires a chi square of 12.59

TABLE A-12 THE PROPORTION MATRIX (P_{jk}) AND SCALE SEPARATION MATRIX (ΔR_{jk}) FROM EXPERIMENT 2.4

P_{jk} =

(N=20)

$C_k \backslash C_j$	C_{50}	C_{44}	C_{39}	C_{34}	C_{28}
C_{50}	.500	.65	.95	1.00	---
C_{44}	.35	.500	.90	.80	.90
C_{39}	.05	.10	.500	.75	.80
C_{34}	0	.20	.25	.500	.65
C_{28}	---	.10	.20	.35	.500

ΔR_{jk} =

$C_k \backslash C_j$	C_{50}	C_{44}	C_{39}	C_{34}	C_{28}
C_{50}	0	.3853	1.6449	---	---
C_{44}	- .3853	0	1.2816	.8416	1.2816
C_{39}	-1.6449	-1.2816	0	.6745	.8416
C_{34}	---	- .8416	- .6745	0	.3853
C_{28}	---	-1.2816	- .8416	- .3853	0
ΔR_j	---	.3600	.8860	.3413	.3444
R_j	0	.3600	1.2460	1.5873	1.9317

Because there are elements of ΔR_{jk} missing, the ΔR_j values are computed as follows:

$$\Delta R_j = \sum^{k} (R_{jk} - R_{j-1,k})/\text{no. of k's}$$

- Chi square for the test of internal consistency: $\chi^2 = 6.89$
- For R_j to be significantly inconsistent with the data at the 0.05 level requires a chi square of 12.59

TABLE A-13 THE PROPORTION MATRIX (P_{jk}) AND SCALE SEPARATION MATRIX (ΔR_{jk}) FROM EXPERIMENT 2.5

P_{jk} = (N=20)

C_k \ C_j	C_{30}	C_{29}	C_{28}	C_{27}	C_{26}	C_{13}
C_{30}	.500	.55	.65	.90	--	--
C_{29}	.45	.500	.55	.75	--	--
C_{28}	.35	.45	.500	.80	.70	.60
C_{27}	.10	.25	.20	.500	.55	.70
C_{26}	--	--	.30	.45	.500	.65
C_{13}	--	--	.40	.30	.35	.500

ΔR_{jk}=

C_k \ C_j	C_{30}	C_{29}	C_{28}	C_{27}	C_{26}	C_{13}
C_{30}	0	.1257	.3853	1.2816	---	---
C_{29}	- .1257	0	.1257	.6745	---	---
C_{28}	- .3853	- .1257	0	.8416	.5244	.2533
C_{27}	-1.2816	-.6745	-.8416	0	.1257	.5244
C_{26}	---	---	-.5744	-.1257	0	.3853
C_{13}	---	---	-.2533	-.5244	-.3853	0
ΔR_j	---	.2795	.0860	.5246	.0183	.2245
R_j	0	.2795	.3655	.8901	.9084	1.1329

Because there are elements of ΔR_{jk} missing, the ΔR_j values are computed as follows:

$$\Delta R_j = \sum^{k} (R_{jk} - R_{j-1,k})/\text{no. of k's}$$

- Chi square for the test of internal consistency: $\chi^2 = 5.71$
- For R_j to be significantly inconsistent with the data, at the 0.05 level requires a chi square of 18.31

TABLE A- 14 THE PROPORTION MATRIX (P_{jk}) AND SCALE SEPARATION MATRIX (ΔR_{jk}) FROM EXPERIMENT 2.6

P_{jk} =

(N= 20)

C_k \ C_j	C_{35}	C_{34}	C_{33}	C_{32}	C_{31}
C_{35}	.500	.80	.70	.90	---
C_{34}	.20	.500	.70	.60	.75
C_{33}	.30	.30	.500	.70	.75
C_{32}	.10	.40	.30	.500	.75
C_{31}	---	.25	.25	.25	.500

ΔR_{jk} =

C_k \ C_j	C_{35}	C_{34}	C_{33}	C_{32}	C_{31}
C_{35}	0	.8416	.5244	1.2816	---
C_{34}	- .8416	0	.5244	.2533	.6745
C_{33}	- .5244	- .5244	0	.5244	.6745
C_{32}	-1.2816	- .2533	- .5244	0	.6745
C_{31}	---	- .6745	- .6745	- .6745	0
ΔR_j	---	.6779	.0921	.3031	.4801
R_j	0	.6779	.7700	1.0731	1.5531

Because there are elements of ΔR_{jk} missing, the ΔR_j values are computed as follows:

$$\Delta R_j = \sum^{k} (R_{jk} - R_{j-1,k})/\text{no. of k's}$$

- Chi square for the test of internal consistency: $\chi^2 = 5.95$
- For R_j to be significantly inconsistent with the data at the 0.05 level requires a chi square of 12.59

TABLE A- 15 THE PROPORTION MATRIX (P_{jk}) AND SCALE SEPARATION MATRIX (ΔR_{jk}) FROM EXPERIMENT 2.7

P_{jk} = (N= 40)

C_k \ C_j	C_{40}	C_{39}	C_{38}	C_{37}	C_{36}
C_{40}	.500	.725	.850	.900	---
C_{39}	.275	.500	.775	.825	.850
C_{38}	.150	.225	.500	.750	.875
C_{37}	.100	.175	.250	.500	.750
C_{36}	---	.150	.125	.250	.500

ΔR_{jk} =

C_k \ C_j	C_{40}	C_{39}	C_{38}	C_{37}	C_{36}
C_{40}	0	.5978	1.0364	1.2816	---
C_{39}	- .5978	0	.7554	.9346	1.0364
C_{38}	-1.0364	- .7554	0	.6745	1.1503
C_{37}	-1.2816	- .9346	- .6745	0	.6745
C_{36}	---	-1.0364	-1.1503	- .6745	0
ΔR_j	---	.4559	.4191	.4498	.4817
R_j	0	.4559	.8750	1.3248	1.8065

Because there are elements of ΔR_{jk} missing, the ΔR_j values are computed as follows:

$$\Delta R_j = \sum^{k} (R_{jk} - R_{j-1,k})/\text{no. of } k\text{'s}$$

- Chi square for the test of internal consistency: $\chi^2 = 8.73$
- For R_j to be significantly inconsistent with the data at the 0.05 level requires a chi square of 12.59

TABLE A- 16 THE PROPORTION MATRIX (P_{jk}) AND SCALE SEPARATION MATRIX (ΔR_{jk}) FROM EXPERIMENT 2.8

P_{jk} =

(N= 20)

C_k \ C_j	C_{45}	C_{44}	C_{43}	C_{42}	C_{41}
C_{45}	.500	.80	.80	.90	---
C_{44}	.20	.500	.75	.95	.95
C_{43}	.20	.25	.500	.50	.90
C_{42}	.10	.05	.50	.500	.75
C_{41}	---	.05	.10	.25	.500

ΔR_{jk} =

C_k \ C_j	C_{45}	C_{44}	C_{43}	C_{42}	C_{41}
C_{45}	0	.8416	.8416	1.2816	---
C_{44}	- .8416	0	.6745	1.6449	1.6449
C_{43}	- .8416	- .6745	0	0	1.2816
C_{42}	-1.2816	-1.6449	0	0	.6745
C_{41}	---	-1.6449	-1.2816	- .6745	0
ΔR_j	---	.3717	.6714	.4035	.6577
R_j	0	.3717	1.0431	1.4466	2.1043

Because there are elements of ΔR_{jk} missing, the ΔR_j values are computed as follows:

$$\Delta R_j = \sum^{k} (R_{jk} - R_{j-1,k})/\text{no. of } k\text{'s}$$

- Chi square for the test of internal consistency: $\chi^2 = 7.7$
- For R_j to be significantly inconsistent with the data at the 0.05 level requires a chi square of 12.59

TABLE A-17 EXPECTED SCALE SEPARATION MATRIX ($\Delta R'_{jk}$) AND EXPECTED PROPORTION MATRIX (P'_{jk}) GIVEN THE RESULTS OF EXPERIMENT 1.1

$\Delta R'_{jk}$ =

C_k \ C_j	C_5	C_1	C_4	C_2	C_3
C_5	---				
C_1	- .513	---			
C_4	- .822	- .309	---		
C_2	-1.354	- .841	- .532	---	
C_3	-1.854	-1.341	-1.032	- .500	---
R_j	0	.513	.822	1.354	1.854

The R_j scale at the bottom comes from Fig. A-1, and the elements of the matrix are computed from it.

P'_{jk} =

C_k \ C_j	C_5	C_1	C_4	C_2	C_3
C_5	---				
C_1	.304	---			
C_4	.206	.381	---		
C_2	.088	.200	.296	---	
C_3	---	.090	.151	.309	---

$$\chi^2 = \frac{N}{821} \sum \left(\mathrm{Sin}^{-1}\sqrt{P_{jk}} - \mathrm{Sin}^{-1}\sqrt{P'_{jk}} \right)^2 = 4.91$$

Where P_{jk} for experiment 1.1 are found in Fig. A-1.

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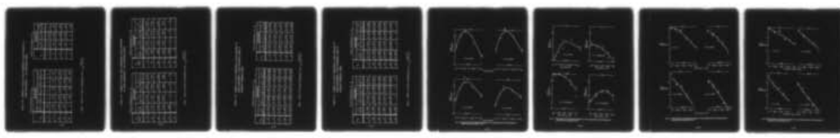

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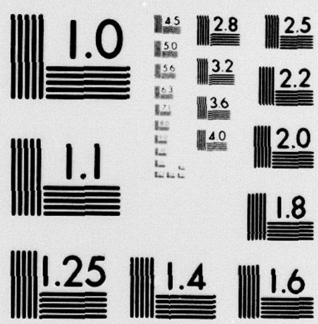

MICROCOPY RESOLUTION TEST CHART

TABLE A-18 SUMMARY OF BASIC PILOT PREFERENCE SCALES PRODUCED BY GRAND EXPERIMENT 1 (NO CONSTANTS ADDED)

Pilot Preference vs G_{ss}

G_{ss}	EXPERIMENT				
	1.1 (E_3=.2)	1.2 (E_3=.3)	1.3 (E_3=.4)	1.4 (E_3=.5)	1.5 (E_3=.6)
6	.513	0	0	0	0
9	1.354	.943	.720	.745	.619
12	1.854	1.550	1.185	1.070	1.529
18	.822	1.098	.813	1.018	1.573
24	0	.151	.324	.580	.766

Pilot Preference vs E_3

E_3	EXPERIMENT		
	1.6 (G_{ss}=6)	1.7 (G_{ss}=12)	1.8 (G_{ss}=24)
.2	1.047	1.5213	0
.3	.832	1.6262	.401
.4	.573	1.7491	.678
.5	.007	1.0959	.831
.6	0	0	.513

Note: Units of Pilot Preference are $\sigma_1 \sqrt{2(1-r_1)}$

TABLE A-19 SUMMARY OF BASIC PILOT PREFERENCE SCALES PRODUCED BY GRAND EXPERIMENT 2 (NO CONSTANTS ADDED)

Pilot Preference vs. E_1

E_1 (SEC)	EXPERIMENT			
	2.1 (E_2=0)	2.2 (E_2=.04)	2.3 (E_2=.08)	2.4 (E_2=.12)
0	1.5562	1.7138	2.2805	1.9317
.04	1.4751	1.4763	1.5064	1.5873
.08	1.3332	1.0010	1.3380	1.2460
.12	.6790	0.8979	.4984	.3600
.16	.4556	0	0	0
.20	0	---	---	---

Pilot Preference vs. E_2

E_2 (SEC)	EXPERIMENT			
	2.5 (E_1=0)	2.6 (E_1=.04)	2.7 (E_1=.08)	2.8 (E_1=.12)
0	1.1329	1.5532	1.8065	2.1043
.04	.9084	1.0731	1.3248	1.4466
.08	.8901	.7700	.8750	1.0431
.12	.3655	.6779	.4559	.3717
.16	.2795	0	0	0
.20	0	---	---	---

Note: Units of Pilot Preference are $\sigma_2\sqrt{2(1-r_2)}$

TABLE A-20 SUMMARY OF PILOT PREFERENCE SCALES PRODUCED BY GRAND EXPERIMENT 1 WITH CONSTANTS ADDED TO MINIMIZE INTERSECT ERROR

Pilot Preference vs. G_{ss}

G_{ss}	EXPERIMENT				
	1.1 (E_3=.2)	1.2 (E_3=.3)	1.3 (E_3=.4)	1.4 (E_3=.5)	1.5 (E_3=.6)
6	11.1741	10.9808	11.0912	10.6891	10.0000
9	12.0151	11.9238	11.8112	11.4341	10.6190
12	12.5151	12.5308	12.2762	11.7591	11.5290
18	11.4831	12.0788	11.9042	11.7071	11.5730
24	10.6611	11.1318	11.4152	11.2691	10.7660

Pilot Preference vs. E_3

E_3	EXPERIMENT		
	1.6 (G_{ss}=6)	1.7 (G_{ss}=12)	1.8 (G_{ss}=24)
.2	11.3422	12.4445	10.5638
.3	11.1272	12.5494	10.9648
.4	10.8682	12.6723	11.2418
.5	10.3022	12.0191	11.3948
.6	10.2952	10.9232	11.0768

Note: Units of Pilot Preference are $\sigma_1\sqrt{2(1-r_1)}$

TABLE A-21 SUMMARY OF PILOT PREFERENCE SCALES PRODUCED BY GRAND EXPERIMENT 2 WITH CONSTANTS ADDED TO MINIMIZE INTERSECT ERROR

Pilot Preference vs. E_1

E_1	EXPERIMENT			
	2.1 (E_2=0)	2.2 (E_2=.04)	2.3 (E_2=.08)	2.4 (E_2=.12)
0	12.7580	12.4338	12.5824	11.9317
.04	12.6769	12.2063	11.8083	11.5873
.08	12.5350	11.7310	11.6399	11.2460
.12	11.8808	11.6279	10.8003	10.3600
.16	11.6574	10.7300	10.3019	10.0000
.20	11.2018	---	---	---

Pilot Preference vs. E_2

E_2	EXPERIMENT			
	2.5 (E_1=0)	2.6 (E_1=.04)	2.7 (E_1=.08)	2.8 (E_1=.12)
0	12.7374	12.6054	12.4796	12.0284
.04	12.5129	12.1253	11.9979	11.3707
.08	12.4946	11.8222	11.5481	10.9672
.12	11.9700	11.7301	11.1290	10.2958
.16	11.8840	11.0522	10.6731	9.9241
.20	11.6045	---	---	---

Note: Units of Pilot Preference are $\sigma_2 \sqrt{2(1-r_2)}$

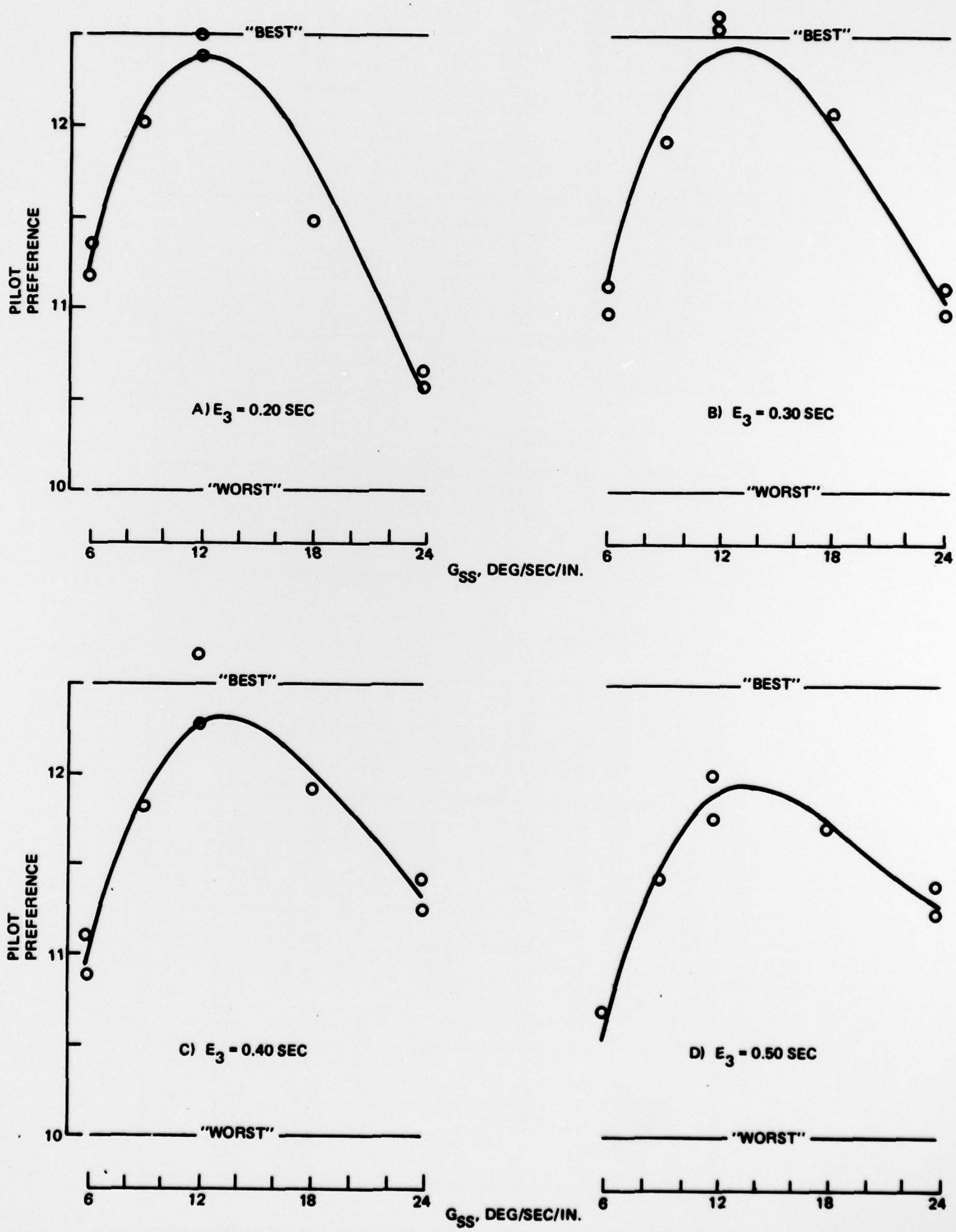

Fig. A-1 Planar Sections of the Polynomial Surface Fit to the Adjusted Data of Grand Experiment 1 Compared to the Data (Sheet 1 of 2)

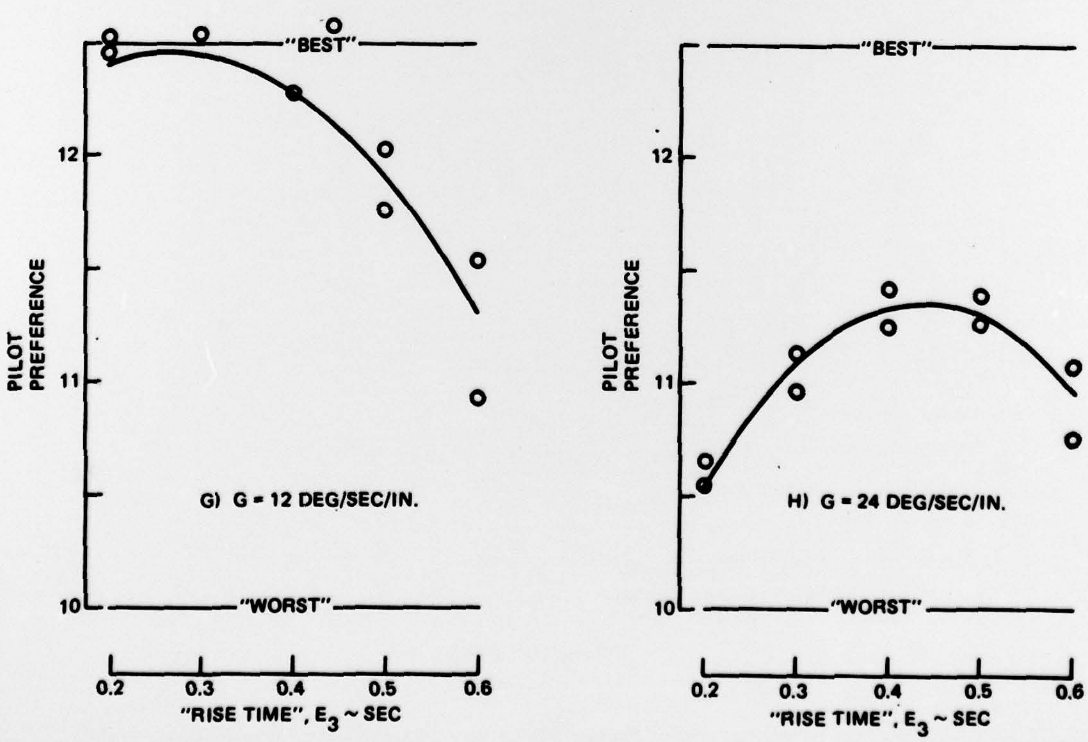

Fig. A-1 Planar Sections of the Polynomial Surface Fit to the Adjusted Data of Grand Experiment 1 Compared to the Data (Sheet 2 of 2)

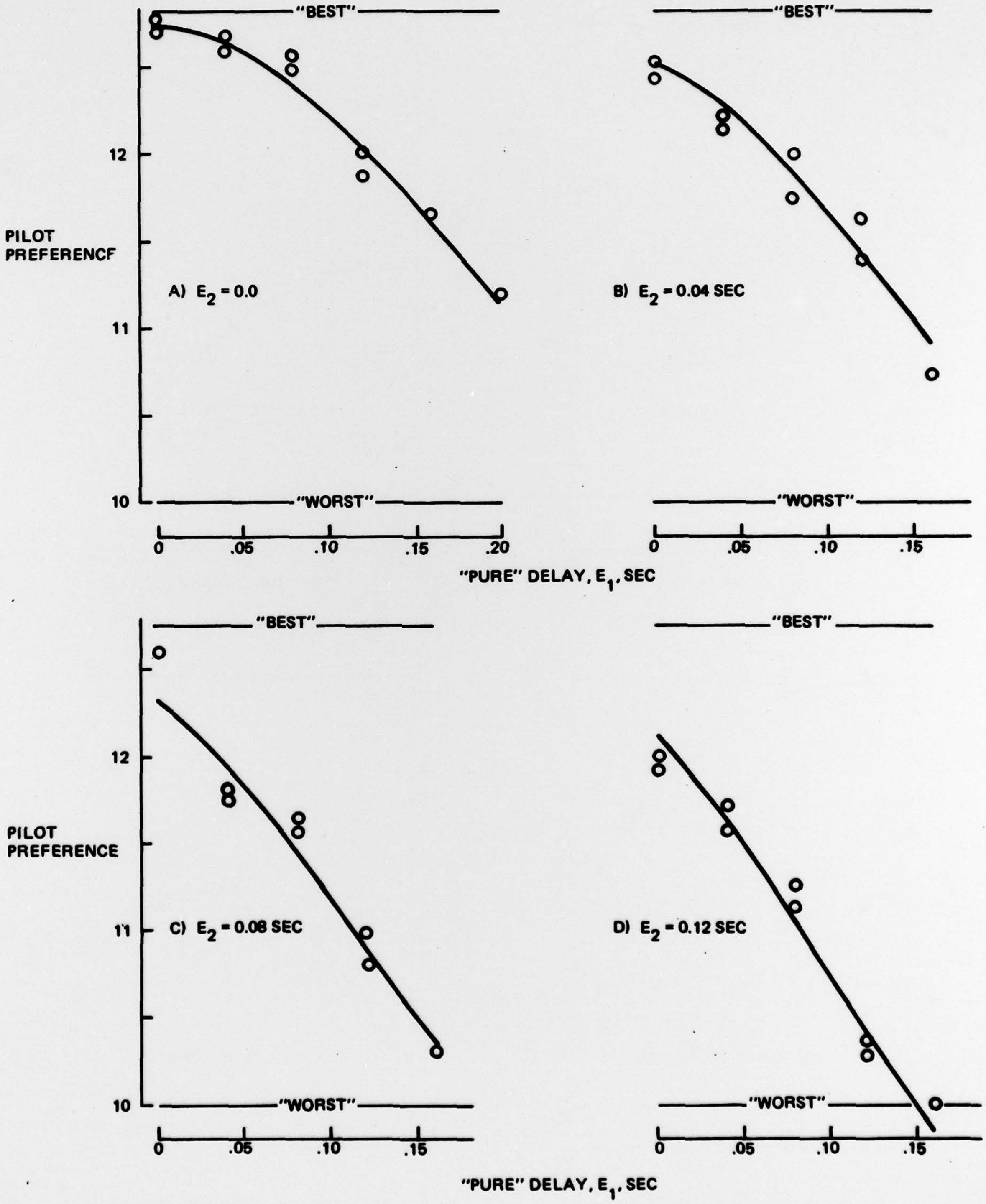

Fig. A-2 Planar Sections of the Polynomial Surface Fit to the Adjusted Data of Grand Experiment 2 Compared to the Data (Sheet 1 of 2)

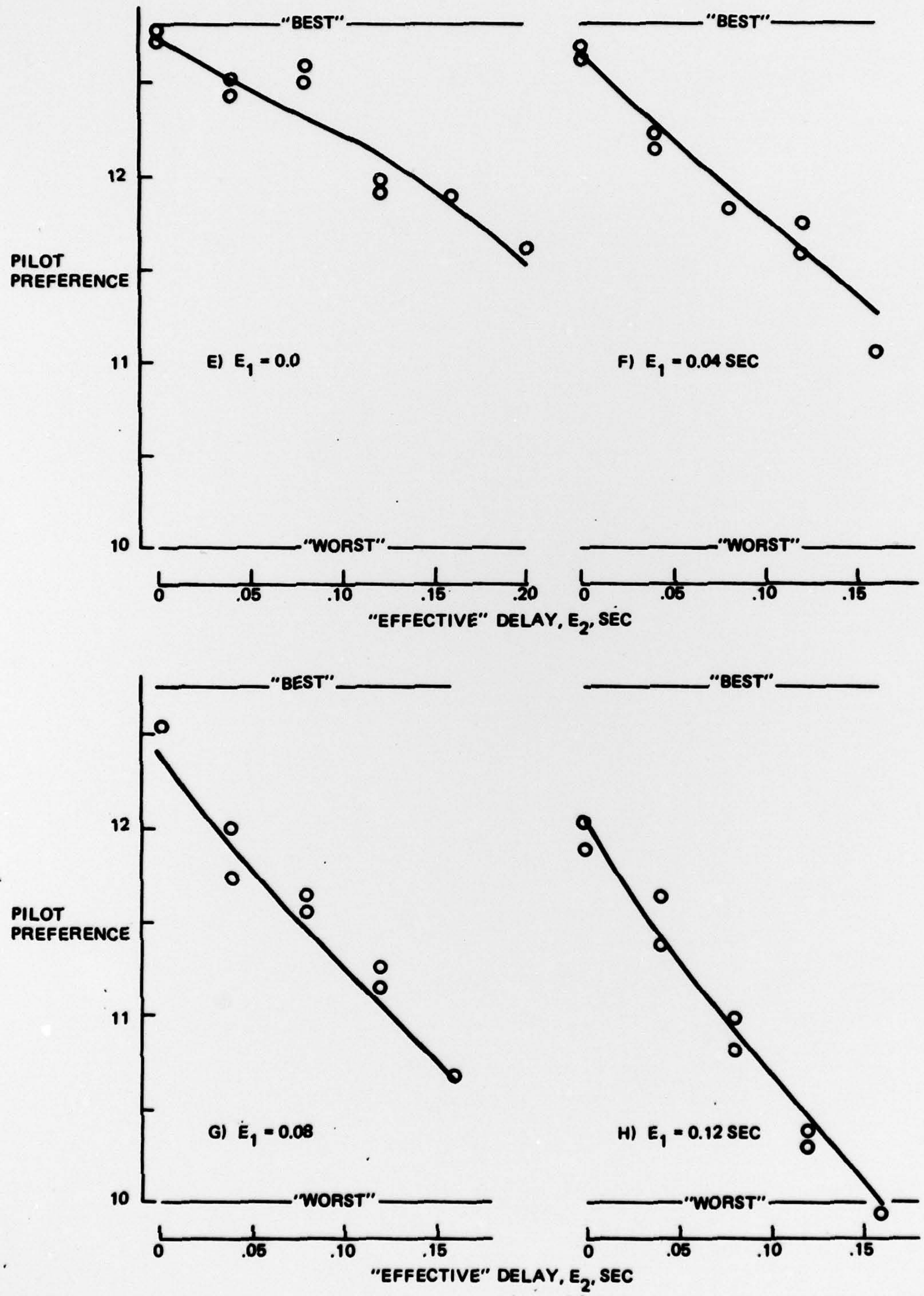

Fig. A-2 Planar Sections of the Polynomial Surface Fit to the Adjusted Data of Grand Experiment 2 Compared to the Data (Sheet 2 of 2)

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